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MULTIMODAL LOGIC FOR SYNTAX

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Abstract
We present a formalization of some ideas from Chomsky [9] in the framework of multimodal type logical grammar (e.g. Morrill [26], Moortgat [24], Cornell [11]).

1. Introduction

1.1. Categorial Grammar[1]

Categorial Grammar is very well known by logicians. It takes place inside a tradition coming from the Polish School, initiated by Łeśnewski, and then followed by Ajduckiewicz [2], Bar-Hillel [3], and culminating in Lambek's famous paper [18] title of which was: "The mathematics of Sentence Structure". During a long period, this trend was dominated by another one, coming from works on formal grammars and initiated by Post and followed by Chomsky, Miller and many other people working mainly in the domain of computer science. It is only in the early eighties that Categorial Grammar was revisited and applied to linguistics, by Steedman (Ades & Steedman, [1]), van Benthem [4], Moortgat [23], Morrill [26] and others. At the end of the eighties, a book collecting many contributions made during a colloquium held in Tucson, Arizona was very seminal (Bach, Oehrle & Wheeler, [28]): it contained a new paper by Lambek ([20]) and numerous applications to the linguistic field. In 1987, Girard published his paper "Linear Logic" ([13]) and it became quite obvious that the Lambek calculus was a fragment of non-commutative intuitionnistic linear logic, limited to multiplicative connectives. Linear Logic, and therefore the Lambek calculus, belongs to the family of "resource-conscious" logics. This means that these logics allow control on the use of formulae. Or, said in

[1] See for an historical introduction: Casadio [7], or Blackburn et al. [5], or Lecomte [21]

other terms, formulae are no longer simple representations of thoughts, but they are viewed exactly like resources. For instance, after a formula has been used as a premise in a proof by the [E]-rule, it can no longer be used, except if this formula was affected by the so-called "exponential" *of course* (or *bang*...) denoted by "!". Van Benthem [4] points out the fact that categorial grammar can be viewed as a logic of *tokens*: a reduction rule like the usual "A/B B → A" is a mere instance of *modus ponens*, provided that we accept the idea that premises disappear after use. In fact, many aspects of syntax in natural languages can be viewed as processes of resource-consumption. Almost all more or less formalized theories of syntax make assumptions on selectional verbal properties. For instance, Lexical Functional Grammar explicitly states that all the complements of a verbal head must be realized as arguments in the functional structure of a sentence, and *only them*. The Chomskyan theory assumes the θ-criterion according to which "every referential expression receives its θ-role from one, and only one θ-position, and every θ-role is assigned to one, and only one θ-position, occupied by a unique referential expression". These restrictions have not to be explicitly introduced if we work inside a framework where they are already encapsulated. A resource-conscious logic provides such a framework. But of course, the pure Lambek calculus suffers from many limitations. It can easily be shown that, for instance, it cannot give any account to non-peripheral extraction in sentences (if we can obviously analyse a sentence like: *Which book did Mary try to read?*, we cannot do it for a sentence like: *Which book did Mary read yesterday?*). Moreover, topics like co-ordination have been much studied in the categorial framework. The solution amounts to assign *polymorphic types* to conjunction words. A polymorphic type can be interpreted as a quantified formula, where the quantification is over variables of types (and thus is a second-order quantification), for instance: (X)(X\X)/X. But such a solution overgenerates and accepts non grammatical phrases like: **(the mathematician) whom Gottlob admired Kazimierz and Jim detested* (Steedman, [33]), because in this sentence, X can be instanciated to the base-type s.

1.2. The Minimalist Program

If Categorial Grammar is well known by logicians, it is probably not the case for the latest developments in Chomsky's theory. We cannot present such developments in details inside the limits of this paper (see Chomsky [9], Pollock [30], Stabler [32]). We shall only restrict ourselves to *Generalized Transformations*, a reminiscence of old transformations that we encountered in the former “Standard Model” of the Generative and Transformational Grammar. In Chomsky [9], the X-bar Theory (which was still a basic component of the previous model) which explained the constituent structure of phrases, is replaced by the GT “Merge”, which builds syntactic objects from more primitive ones. For instance, by Merge, *speaks* and *French* form a new syntactic object *speaks French*. We must only indicate what is the head of this new object. We have the choice between *speaks* and *French*, but here, because this object acts again as a verbal element, *speaks* is the head. When this object is obtained, it can be merged again, this time with some new nominal phrase, say *Peter*, in such a way that we get *Peter speaks French*, with again *speaks* as the head. But of course, this is not the whole story because one of the striking things in human languages is that “objects appear in the sensory output in positions ‘displaced’ from those in which they are interpreted” (Chomsky [9], p. 222). And, as Chomsky says “these displacement properties are one central syntactic respect in which natural languages differ from the symbolic systems devised for one or another purpose, sometimes called ‘languages’ by metaphoric extension (formal languages, programming languages)”. An example of displacement is what we observe in questions like: *Which book do you read?* where the wh-phrase *which book* is displaced from an original site, which is just after the verb. In French, we should have: *Quel livre lis-tu?* but in a more popular usage we could perfectly have: *Tu lis quel livre?* In other languages like Chinese, the wh-phrase is not moved. These languages are called *in situ* for this reason. The wh-displacement is therefore not something motivated by some reason of *meaning*, it looks like some arbitrary choice made by languages. Another example is the placement of adverbs in French and in English. Whereas we have in English: *Peter tenderly kisses Mary*, we have in French: *Pierre embrasse tendrement Marie*. What compels the syntax of each language to choose one verb-adverb order preferably to the other? It is not a question of meaning. The solution proposed by Chomsky resides in some (morphological) features that lexical and syntactical objects have, and which must be necessarily checked in the course of one derivation. These features may be *weak* or else *strong*. Checking a strong feature amounts to displacing the phonological part of an object together with the feature which must be checked. Checking a weak feature only amounts to displacing the feature (and possibly the semantical part of the object, but not the phonological one). Checking results in a displacement because if the feature cannot be checked *in situ*, it must be moved in order to be checked in another position, a position where it can be. Such positions are, technically speaking, corresponding to the specifier positions of heads. Thus, moves take place towards these specifier positions. For instance, the target of a wh-expression in case of a non *in situ* language is the specifier of the COMP-head. Moreover, verbal heads (like *kisses* or *embrasse*) are subject to displacements when the language has a rich verbal morphology, which is the case for Romance languages. This explains why in French verbal heads precede adverbials (they are attracted by the node “inflection”). To summarize, the *Move* transformation is triggered by feature-checking.

1.3. Convergence

Many authors have already noticed the convergence between the two programs: Categorial Grammar on one side, and Minimalism on the other (Berwick & Epstein [5], Stabler [32], Lecomte [22]). The most obvious convergence point concerns the operation *Merge* which is quite similar to the cancellation schemes of very traditional Ajduckiewicz-Bar-Hillel grammars. For *speaks* and *French*, it is enough to assign the type **(np\s)/np** to *speaks* and the type **np** to *French*, the resulting objects associated with *speaks French* will be of type **np\s**. The headness of *speaks* will be marked by the common base-category **s** which belongs to *speaks* as well as to *speaks French*.

But it has always been uneasy to express the Generalized Transformation *Move* in the categorial frame. However, the enrichment of this frame made during the last ten years (essentially by the way of introducing *structural modalities*) now enables us to give an account of displacements and of the role of features in their occurrence. This is what we shall try to show in the following sections.

2. The Grammar Logic

*2.1. The basic **NL***

We shall use a sequent presentation. Rules are given as inference steps allowing us to infer a deduction relation expressed by an intuitionistic “sequent”: $\Gamma \vdash A$ (where Γ is a structured multiset of formulae and A is a formula) from other ones (actually one another in the case of unary rules and two others in the case of binary rules). More precisely, we shall note $\Delta[X]$ a structured multiset of formulae in which there is a distinguished occurrence of X; subsequent uses of $\Delta[Y]$ meaning the same structure but with Y in place of X. We shall work inside a variant of the Lambek calculus, called **NL** (non-associative Lambek calculus (Lambek [19])).

The rules of **NL** are very well known but we give them below.

- identity axiom

$$A \vdash A$$

- Left introduction of / and \

$$[L/]\ \frac{\Theta \vdash B \qquad \Gamma[A] \vdash C}{\Gamma[(A / B, \Theta)] \vdash C} \qquad [L\backslash]\ \frac{\Theta \vdash B \qquad \Gamma[A] \vdash C}{\Gamma[(\Theta, B \backslash A)] \vdash C}$$

- Right introduction of / and \

$$[R/]\ \frac{(\Gamma, B) \vdash A}{\Gamma \vdash A / B} \qquad [R\backslash]\ \frac{(B, \Gamma) \vdash A}{\Gamma \vdash B \backslash A}$$

- Product

$$[L\bullet]\ \frac{\Gamma[(A, B)] \vdash C}{\Gamma[A \bullet B] \vdash C} \qquad [R\bullet]\ \frac{\Gamma \vdash A \quad \Delta \vdash B}{(\Gamma, \Delta) \vdash A \bullet B}$$

- Cut

$$\frac{\Theta \vdash A \qquad \Gamma[A] \vdash B}{\Gamma[\Theta] \vdash B}$$

*Examples of proofs in **NL**:*

John considers Mary deaf (from Morrill [26])

with the type assignment:
John: N
Mary: N
deaf: CN/CN
considers: (N\S)/(N • (CN/CN))
The sequent to prove is:

$$N, (N\backslash S)/(N \bullet (CN/CN)), N, CN/CN \vdash S$$

(with a certain parenthetisation to find)

The proof is:

$$\dfrac{\dfrac{N \vdash N \qquad \dfrac{\dfrac{CN \vdash CN \qquad CN \vdash CN}{(CN/CN\ CN) \vdash CN}}{CN/CN \vdash CN/CN}}{(N, CN/CN) \vdash N \bullet CN/CN} \qquad \dfrac{N \vdash N \qquad S \vdash S}{(N, (N\backslash S)) \vdash S}}{(N, ((N\backslash S)/(N \bullet (CN/CN)), (N, CN/CN))) \vdash S}$$

*Example of failure in **NL**:*

the man that John saw

with the type assignment:
the: N/CN
man: CN
that: (CN\CN)/(S/N)
John: N
saw: (N\S)/N

we try the proof:

$$\dfrac{\dfrac{((N, (N\backslash S)/N), N) \vdash S}{(N, (N\backslash S)/N) \vdash S/N} \qquad \dfrac{CN \vdash CN \qquad (N/CN, CN) \vdash N}{(N/CN, (CN, CN\backslash CN)) \vdash N}}{N/CN, (CN, (CN\backslash CN)/(S/N), (N, (N\backslash S)/N)) \vdash N}$$

but it fails because ((N, (N\S)/N), N) ⊢ S is not provable. Notice however that this sequent is provable in **L** (the Lambek calculus with associativity). Here, we continue the proof by explicitly assuming associativity, as a structural rule:

$$\begin{array}{c} \Gamma[(A, (B, C))] \vdash D \\ \hline\hline \Gamma[((A, B), C)] \vdash D \end{array}$$

(where the double bar means that the rule can be applied in both directions)

$$\begin{array}{c} \dfrac{N \vdash N \qquad \dfrac{N \vdash N \qquad S \vdash S}{(N, N\backslash S) \vdash S}}{(N, ((N\backslash S)/N, N)) \vdash S} \\ \hline\hline ((N, (N\backslash S)/N), N) \vdash S \end{array}$$

2.2. The modals

As we saw on the previous example, **NL** is very weak! One way of enriching it is to allow a local use of the structural rule of associativity when necessary. This amounts to consider new connectives, but unary ones, called *structural modalities* (similar in their essence to the exponentials of Linear Logic, in the sense that they allow not only a restriction on the use of resources but a control on it). Structural modalities are very similar to modalities in Modal Logic in that they obey similar introduction laws and they accept an interpretation in terms of Possible Worlds. But their name comes from the fact that they are subject to some specific structural rules. For instance, it is easy to assume that in **NL**, whenever a subtree contains a modalized type, it is possible to use associativity, and only in such a case. In this case, the structural modality will be used in order to give local associativity to the system. Let us, for instance, assume that, in the previous example, the complementizer *that* is assigned the type (CN\CN)/(S/□N) instead of (CN\CN)/(S/N). Then we should have had the following deduction:

$$
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\dfrac{N \vdash N \qquad \dfrac{N \vdash N \qquad S \vdash S}{N, N\backslash S \vdash S}}{(N, ((N\backslash S)/N, N)) \vdash S}
}{(N, ((N\backslash S)/N), \Box N)) \vdash S}
}{((N, (N\backslash S)/N), \Box N) \vdash S}
}{(N, (N\backslash S)/N) \vdash S/\Box N}
\qquad
\dfrac{CN \vdash CN \qquad (N/CN, CN) \vdash N}{(N/CN, (CN, CN\backslash CN)) \vdash N}
}{N/CN, (CN, (CN\backslash CN)/(S/\Box N), (N, (N\backslash S)/N) \vdash N}
$$

where the important point to notice is that by assigning to *that* the type (CN\CN)/(S/□N) i.e. some type which asks for a S/□N, when we try to prove that something is of type S/□N, this compels us to use the [/R] rule, thus introducing inside the antecedent of a sequent a modalized formula □N, which will be able to trigger the associativity rule. We can say that this modality "□" makes **NL** communicate with **L**. The rules are:

$$
[\Box L]\ \dfrac{\Gamma[A] \vdash B}{\Gamma[\Box A] \vdash B} \qquad [\Box R]\ \dfrac{\Box\Gamma \vdash A}{\Box\Gamma \vdash \Box A}
$$

where □Γ means that all the formulae in Γ are modalized.

2.3. Interpretation

As showed by Dosen [12], if we take the language of the **NL-**formulae, defined by:
F::= A | *F/F* | *F* • *F* | *F\F*, where A is a set of atomic formulae (proposition letters), the most general interpretation for such a language is given in terms of Kripke style relational structures: ternary relational structures <W, R^3> in the case of binary connectives. W is to be understood as the set of linguistic resources and R, the accessibility relation, as representing linguistic composition. A valuation in such a frame is a function V from basic types to subsets of W, extended by the following definitions:

V(A • B) = {z;   x y [Rzxy & x   V(A) & y   v(B)]}
V(C/B) = {x;   y   z [(Rzxy & y   V(B))   z   V(C)]}
V(A\C) = {y;   x   z [(Rzxy & x   V(A))   z   V(C)]}

As also recalled in Kurtonina & Moortgat [17], with no restriction on R, we obtain the *pure logic of residuation*, equivalent to **NL**, which can be simply expressed by[2]:

A → C/B   ⇔   A • B → C   ⇔   B → A\C

L is obtained by adding a structural postulate:

(xyz   W),   t (Rxyt & Rtzu) ⇔   v (Rvyz & Rxvu)   (Ass.)

LP (the permutationally free Lambek calculus) is obtained by adding moreover the postulate:

(xyz   W) Rxyz ⇔ Rxzy   (Com.)

Dosen proved the completeness results for these interpretations. These results can be extended to the systems with structural modalities. In this case, we use ternary frames <W, R^2, R^3>, where R^2 is an accessibility relation associated with the unary connective.

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[2] An axiomatic presentation for **NL** can be easily given. The axioms are:
(REFL) A → A
(TRANS) if A → B and B → C then A → C
(RES-L) A • B → C iff A → C/B
(RES-R) A • B → C iff B → A\C

But whereas R^3xyz means "x is the combination of y and z", R^2xy means that x *is accessible from the linguistic resource* y by means of some "building instructions referred to by R^2" (Moortgat [24]). If we extend our interpretation to the formulae of a language extended with modalities, we shall get two dual modalities: $\Box$ and $\Diamond$ such that:

$$V(\Diamond A) = \{x: \ y\ (R^2xy\ \&\ y\ \ V(A)\}$$
$$V(\Box A) = \{x; \ \ y(R^2yx \ \ \ \ y\ \ V(A)\}$$

and we obtain an extension of the logic of residuation[3], which is expressible by:

$$\Diamond A \rightarrow B \quad \Leftrightarrow \quad A \rightarrow \Box B$$

If we interpret $\Diamond A$ as meaning: "A *with* the structure associated with $\Diamond$" and $\Box A$ as meaning: "something which *demands* the corresponding structure in order to become an A", we are led to the "structural" interpretation of modalities we are interested in. And for instance we understand well the theorem $\Diamond\Box A \rightarrow A$, which says that if a linguistic resource is asking for a structure S to become an A and if it receives this structure, then *it is* an A. It is easy to show (Moortgat [24], Dosen [12]) the following completeness result:

Theorem: **NL**$\Diamond$ is sound and complete with respect to the frame semantics. Or in other terms, $A \rightarrow B$ is a theorem of NL$\Diamond$ iff $V(A) \ \ V(B)$ for all valuation V on a ternary frame.

2.4. Modes of composition

Antecedents of sequents in **NL** are in fact binary trees. In working in **NL**, we thus assume a *single* mode of composition for linguistic resources. But many observations lead us to the assumption of *multi*-modality: linguistic objects have several dimensions along which they are composed (Oehrle [27]). Moreover, it seems that we sometimes need more elaborate notions than single binary trees. Modern syntax gives an important role to the notion of *head*. We can assume that at each step of the merging process of syntactic objects, we need to indicate what is the head, thus distinguishing between *two* products: one noted $\bullet_l$ and the other $\bullet_r$, such that $A \bullet_l B$ indicates a left-headed

[3] We obtain the calculus **NL**$\Diamond$ whose axioms are the same as those of **NL**, but with the supplementary one: (RES-1) $\Diamond A \rightarrow B$ iff $A \rightarrow \Box B$.

structure and $A \bullet_r B$ a right-headed one (Moortgat & Morrill [25]). These two products then give us a good example of *two modes of composition*.

We can have in fact several products and their residuates in order to formalize composition modes. Of course, we can assume that they do not obey the same laws. The interpretation of such a mixed system is provided by a multimodal frame $<W, \{R_i^3\}_{i \;\; I}>$ where I is the set of *modes*. There is nothing really new with regards to our previous frames except that from now on we can express more, simply by adding new postulates which make the composition modes communicate. These are *inclusion* postulates and *interaction* postulates. An inclusion postulate has the form: $A \bullet_i B \rightarrow A \bullet_j B$

Interaction postulates give access to restricted forms of structural rules. An example is the following axiom MA (*Mixed Associativity*):

$$\text{(MA)} \qquad A \bullet_i (B \bullet_j C) \leftrightarrow (A \bullet_i B) \bullet_j C$$

In this case, you can exchange the mode i against the mode j, as the main operator, provided that you change the structure of the binary tree. Thus, associativity is allowed only when this exchange is possible. Another example is Restricted Contraction; the following interaction postulate MC gives a restricted access to Contraction:

$$\text{(MC)} \ (A \bullet_i B) \bullet_j C \rightarrow (A \bullet_j C) \bullet_i (B \bullet_j C)$$

Of course, the same game can be played with unary connectives. We can work in a system where several pairs $(\Diamond_{\alpha,} \Box_\alpha)_{\alpha \;\; A}$ are coexisting, which means: several different structure building operations. And there also, we shall be able to define inclusion and interaction postulates between them. For instance, the two following axioms express weak distributivity:

$$\text{(K1)} \ \Diamond(A \bullet B) \rightarrow \Diamond A \bullet B$$
$$\text{(K2)} \ \Diamond(A \bullet B) \rightarrow A \bullet \Diamond B$$

Let us assume, for instance, that $\Diamond$ denotes some feature that a syntactic object can receive. (K1) says that this feature is transmitted to the first conjunct and (K2) says that it is transmitted to the second one. The following axiom (K) says that it is transmitted to both. Let us immediately remark that (K) is the appropriate axiom for expressing the properties of a system of agreement[4].

$$\text{(K)} \quad \Diamond(A \bullet B) \rightarrow \Diamond A \bullet \Diamond B$$

Interpretation can be given to these axioms in terms of the multimodal ternary frames. Each pair $(\Diamond_{\alpha,} \Box_\alpha)$ is associated with some accessibility

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[4] See Heylen, D [14].

relation (to a specific structure) R_α, in the same way that each product $\bullet_i$ is associated with some ternary relation expressing the way of building an x from two objects y and z.

2.5. Sequent representation

The sequent presentation of a mixed calculus requires that the antecedent of each sequent reflect the composition modes and the particular structures associated with the products and modalities which are involved in its construction. For each product $\bullet_i$, there is a structure-building binary operation $(,)^i$ associated with it, and for each modality $\Diamond_\alpha$, there is a structure-building unary operation $( )^{\Diamond\alpha}$ associated with it. Axioms of residuation are converted into the following Gentzen rules.

- <u>identity axiom</u>

$$A \vdash A$$

- <u>Left introduction of / and \</u>

[L/i] $$\frac{\Theta \vdash B \qquad \Gamma[A] \vdash C}{\Gamma[(A /_i B, \Theta)^i] \vdash C}$$ [L\i] $$\frac{\Theta \vdash B \qquad \Gamma[A] \vdash C}{\Gamma[(\Theta, B \backslash_i A)^i] \vdash C}$$

- <u>Right introduction of / and \</u>

[R/i] $$\frac{(\Gamma, B)^i \vdash A}{\Gamma \vdash A /_i B}$$ [R\i] $$\frac{(B, \Gamma)^i \vdash A}{\Gamma \vdash B \backslash_i A}$$

- <u>Product</u>

[L•i] $$\frac{\Gamma[(A, B)^i] \vdash C}{\Gamma[A \bullet_i B] \vdash C}$$ [R•i] $$\frac{\Gamma \vdash A \qquad \Delta \vdash B}{(\Gamma, \Delta)^i \vdash A \bullet_i B}$$

- <u>Cut</u>

$$\frac{\Gamma \vdash A \qquad \Delta[A] \vdash B}{\Delta[\Gamma] \vdash B}$$

• Modalities

$$[\Box_\alpha L]\quad \frac{\Gamma[A] \vdash B}{\Gamma[(\Box_\alpha A)^{\Diamond\alpha}] \vdash B} \qquad [\Box_\alpha R]\quad \frac{(\Gamma)^{\Diamond\alpha} \vdash B}{\Gamma \vdash \Box_\alpha A}$$

$$[\Diamond_\alpha L]\quad \frac{\Gamma[(A)^{\Diamond\alpha}] \vdash B}{\Gamma[\Diamond_\alpha A] \vdash B} \qquad [\Diamond_\alpha R]\quad \frac{\Gamma \vdash A}{(\Gamma)^{\Diamond\alpha} \vdash \Diamond_\alpha A}$$

3. Weak and strong modes and switching from mode to mode

3.1. Weak and strong modes

We call a *strong* mode every couple of modalities ($\Box_\alpha$, $\Diamond_\alpha$) such that the cancellation given by $\Diamond_\alpha\Box_\alpha A \rightarrow A$ can occur only after several steps of *tree restructuring* (using permutation and associativity) and we call a *weak* mode every couple ($\Box_\beta$, $\Diamond_\beta$) such that this cancellation can occur without such restructuring and simply by means of the usual rules of distributivity (K1, K2, K). When a mode is strong, it can be cancelled only by becoming weak, as only the weak mode enjoys distributivity. It is therefore by reaching a weak mode from the corresponding strong one that restructuring is triggered. In order to allow an indefinite number of steps in such a restructuring, we introduce an intermediary mode, which will be said *neutral*. What happens is the following: when going from strong to neutral, a permutation is performed (see the [SN]-rule), but when going from neutral to strong, the following interaction postulate is applied:

$$\Diamond^N((A \bullet B) \bullet C) \rightarrow (A \bullet \Diamond^S(C \bullet B))$$

which combines associativity, commutativity and weak distributivity (K2). The result is that, after restructuring the tree, the strong modality appears lower in the tree, and of course ready for a new cycle. The exchange of modalities strong/neutral/strong/neutral/... necessarily terminates. But, in a correct deduction, it terminates by a last exchange “neutral against weak”, which is simply expressed by a distributivity law. Finally, distributivity postulates are used to eliminate the modalities (= the features). If the deduction is correct, types, stripped from their modalities, stand in the right positions where they can be cancelled by

the reduction rule [L/]. Finally, accepted sentences are those for which there exists a correct deduction.
Let us see here an example. We want to prove the acceptability of the very simple question in French: *qui Pierre connaît?* (*who does Peter know?*) by assuming a strong mode ($\Box^S_{wh}$, $\Diamond^S_{wh}$). We also assume the corresponding neutral and weak corresponding modes: ($\Box^N_{wh}$, $\Diamond^N_{wh}$) and ($\Box^W_{wh}$, $\Diamond^W_{wh}$). We assume the following interaction postulates:

[SN]	$\Diamond^S(A \bullet B) \rightarrow \Diamond^N(B \bullet A)$
[NS]	$\Diamond^N((A \bullet B) \bullet C) \rightarrow (A \bullet \Diamond^S(C \bullet B))$
[NW]	$\Diamond^N((A \bullet B) \bullet C) \rightarrow (A \bullet \Diamond(C \bullet B))$
[Incl]	$\Diamond^N A \rightarrow \Diamond A$
[K1]	$\Diamond(A \bullet B) \rightarrow A \bullet \Diamond B$
[K2]	$\Diamond(A \bullet B) \rightarrow A \bullet \Diamond B$

and we assume the following assignment:

Pierre	:=	$\Box_k$ **np**
connaît	:=	**(vp/np)/np**
qui	:=	$\Box_{wh} \Box_k$ **np**

A part of the deduction is:

....

$$\frac{\dots}{(Pierre, (connaît, \Box_k \mathbf{np})) \vdash \mathbf{vp}}$$

$$\frac{(Pierre, (connaît, \Box_k \mathbf{np})) \vdash \mathbf{vp}}{(Pierre, (connaît, (\Box_{wh} \Box_k \mathbf{np})^{\Diamond wh})) \vdash \mathbf{vp}}\ [L\Box]$$

$$\frac{(Pierre, (connaît, (\Box_{wh} \Box_k \mathbf{np})^{\Diamond wh})) \vdash \mathbf{vp}}{(Pierre, (connaît, (qui)^{\Diamond wh})) \vdash \mathbf{vp}}\ [lex]$$

$$\frac{(Pierre, (connaît, (qui)^{\Diamond wh})) \vdash \mathbf{vp}}{(Pierre, ((connaît, qui))^{\Diamond, wh}) \vdash \mathbf{vp}}\ [K2]$$

$$\frac{(Pierre, ((connaît, qui))^{\Diamond, wh}) \vdash \mathbf{vp}}{(Pierre, ((connaît, qui))^{\Diamond N, wh}) \vdash \mathbf{vp}}\ [Incl]$$

$$\frac{(Pierre, ((connaît, qui))^{\Diamond N, wh}) \vdash \mathbf{vp}}{((Pierre, ((qui, connaît))^{\Diamond S, wh})) \vdash \mathbf{vp}}\ [SN]$$

$$\frac{((Pierre, ((qui, connaît))^{\Diamond S, wh})) \vdash \mathbf{vp}}{(((Pierre, connaît), qui))^{\Diamond N, wh} \vdash \mathbf{vp}}\ [NS]$$

$$\frac{(((Pierre, connaît), qui))^{\Diamond N, wh} \vdash \mathbf{vp}}{\quad}\ [SN]$$

$$\frac{((qui,\ (Pierre,\ connaît)))^{\Diamond S,wh} \vdash \mathbf{vp}}{(qui,\ (Pierre,\ connaît)) \vdash \Box^{S}_{wh}\ \mathbf{vp}}\ [R\Box]$$

This part of the deduction explains how the interrogative *qui* can find back “the position where it is interpreted”, which means the object position where it can be cancelled against the verbal fonctorial type. But of course it remains to explain how the deduction can continue (always in the bottom-up direction).

3.2. Switching the modes

The first time a mode is used in the application of a structural postulate [NS] or [NW] or K1 or K2, we can assume that it already installs the following mode by means of which the deduction will be continued. This amounts in the previous deduction to slightly changing the application of the lowest [NS] rule. The step is in fact:

$$\frac{((Pierre,\ ((qui,\ connaît))^{\Diamond S,wh}))^{\Diamond S,k} \vdash \mathbf{vp}}{(((Pierre,\ connaît),\ qui))^{\Diamond N,wh} \vdash \mathbf{vp}}\ [NS]$$

in such a way that when we arrive at the last step, we get

$$(Pierre,\ (connaît,\ \Box_k\ \mathbf{np}))^{\Diamond S,k} \vdash \mathbf{vp}$$

instead of

$$(Pierre,\ (connaît,\ \Box_k\ \mathbf{np})) \vdash \mathbf{vp}$$

In this way, the deduction can go on: *Pierre*, which is a $\Box_k$ **np**, is displaced by means of [SN], [NS] and finally we get:

$$((connaît,\ \Box_k\ \mathbf{np}),\ (Pierre)^{\Diamond k}) \vdash \mathbf{vp}$$

which succeeds because it gives:

$$((connaît,\ \Box_k\ \mathbf{np}),\ (\Box_k\ \mathbf{np})^{\Diamond k}) \vdash \mathbf{vp}$$

by the [lex]-rule, and:

$$((connaît,\ \Box_k\ \mathbf{np}),\ \mathbf{np}) \vdash \mathbf{vp}$$

by the [L☐]-rule.

But at this step, another mode-switching is needed because there still remains one modality to drop, which corresponds to the objective case. In order that this new mode be installed, it is therefore necessary that $\Diamond^{S}_{k}$ installs the following mode, corresponding to the objective case, but a weak case: $\Diamond_k$. We have the step:

$$\frac{((\textit{connaît}, \Box_k \mathbf{np}), (\textit{Pierre})^{\Diamond k})^{\Diamond k} \vdash \mathbf{vp}}{((\textit{connaît}, \Box_k \mathbf{np}), \textit{Pierre})^{\Diamond k} \vdash \mathbf{vp}}\ [K2]$$

in such a way that we have to prove:

$$((\textit{connaît}, \Box_k \mathbf{np}), (\textit{Pierre})^{\Diamond k})^{\Diamond k} \vdash \mathbf{vp}$$

which is proved because the following sequents are proved:

$$((\textit{connaît}, \Box_k \mathbf{np}), \mathbf{np})^{\Diamond k} \vdash \mathbf{vp}$$
$$(((\textit{connaît}, \Box_k \mathbf{np}))^{\Diamond k}, \mathbf{np}) \vdash \mathbf{vp}$$
$$((\textit{connaît}, (\Box_k \mathbf{np})^{\Diamond k}), \mathbf{np}) \vdash \mathbf{vp}$$
$$((\textit{connaît}, \mathbf{np}), \mathbf{np}) \vdash \mathbf{vp}$$

In order to provide such an amelioration to our system, we must therefore duplicate each mode so that we have two instances of each one: one for the "first" use (or "bottom" use), and the other for the standard uses along the derivation. Let us define then for each α A (A the set of all modes) the following set of pairs of modalities:

$$(\Box^{S0}_{\alpha}, \Diamond^{S0}_{\alpha}), (\Box^{S}_{\alpha}, \Diamond^{S}_{\alpha}), (\Box^{N0}_{\alpha}, \Diamond^{N0}_{\alpha}), (\Box^{N}_{\alpha}, \Diamond^{N}_{\alpha}),$$
$$(\Box^{0}_{\alpha}, \Diamond^{0}_{\alpha}), (\Box_{\alpha}, \Diamond_{\alpha})$$

And let us also assume a total order between modes, such that *if* β *is the successor of* α, then the $(\Box^{N0}_{\alpha}, \Diamond^{N0}_{\alpha})$ and the $(\Box^{0}_{\alpha}, \Diamond^{0}_{\alpha})$ modalities *give access to* a $(\Box^{0}_{\beta}, \Diamond^{0}_{\beta})$ - system (strong or weak). We do not display the resulting communication rules [S_0N_0], [N_0S], [N_0W], [K_01], [K_02] which all come from the previous ones in a natural way.

In conclusion of this section, let us point out a feature of the architecture that we suggest. Besides "logical" axioms like the previous postulates, the spefically grammatical part of the system is provided by two kinds of information:

1- *a hierarchy of modes*, which is cross-linguistically invariant (and which reflects in many respects the hierarchy of functional nodes in the Chomskyan structure of the sentence),

2- *a choice between Strong and Weak for each mode*, which is specific to any language.

The first kind of information belongs to the so-called "Principles" of the linguistic theory, and the second one to the so-called "Parameters".

4. Agreement and inflection

4.1. The problem

In the modern theory of syntax, the sentence is the projection of the Inflection head. This translates into a node IP (or I'') which dominates a specifier (Spec, I') and an intermediary projection I' which itself dominates the inflection head I_0. The subject nominal phrase migrates towards (Spec, I') because it is there that it can receive its nominative case (and according to the Case-Filter, every nominal phrase must receive a case). For that reason, we use the strong mode of composition ($\Box^{S0}_{knom}$, $\Diamond^{S0}_{knom}$), called by ($\Box^{N0}_{wh}$, $\Diamond^{N0}_{wh}$), the effect of which is to take the first element dominated by the modality, which is necessarily the subject because of the aforementioned principle, and to move it backwards to the position where it will be cancelled by the verb.
Chomsky sometimes changes the label IP into AGRP, and I_0 becomes AGR_0. AGR_0 is then used to manage the subject-verb agreement[5]. The agreement features of the NP subject and those of the verbal head must be checked *before* the NP subject moves. This means that between the mode of composition *wh* and the mode k_{nom}, must be inserted a mode *agr*, which is responsible for the subject-verb agreement. When the $\Diamond$-modality *agr* reaches the verbal phrase, it splits into two modalities: the second part of the *agr* modality, *agrV*, in order to check the agreement features of the verb (number, person) and the *infl* modality which will be responsible for the tense feature checking. At this precise point, there can be a divergence between languages. Either this *infl* modality is strong and this results in a language where the verbal head must be the nearest as possible to the subject (thus giving the adverb following the verb like in French) or it is weak and this results in no verbal head move, (thus giving the adverb preceding the verb like in English).
We shall therefore introduce special interaction postulates for *agr* and *infl*, with the assumption that *wh* precedes *agr*, which precedes *infl*, which precedes k_{nom}

$$[K_{agr}]: \quad \Diamond^0_{agr}(A \bullet B) \rightarrow \Diamond_{agrN}A \bullet \Diamond^0_{agrV}B$$
$$[\text{Agr-Infl}]: \quad \Diamond^0_{agrV}(A \bullet B) \rightarrow \Diamond^{*0}_{infl}(\Diamond_{agrV}A \bullet B)$$

[5] More precisely, Chomsky [9] assumes *two* agreement nodes: one for the subject-verb agreement AGR^S and another one for the object-verb agreement AGR^O. We shall not speak of the latter in this paper.

where the "*" still denotes either S (strong) or W (weak) according to the language[6]. This implies that verbs are assigned modalized types. For instance, a tensed verb is assigned the type:

$$\Box_{agrV}\Box_{infl}\ V$$

where V is an abbreviation for **vp/np**, **(vp/np)/np**, **(vp/np)/pp** …

Moreover, the modes of composition *agr* and *infl* are actually represented by *families of modalities*. In ($\Box^0_{agrV}$, $\Diamond^0_{agrV}$) for instance *agrV* must be seen as a variable which must be instantiated by a particular set of feature value pairs. These pairs are those which are relevant for the verb agreement (in French and in English, person and number, but not gender for instance[7]). The same thing is required for agr_N (the relevant features being for instance in French person, number and gender) and for *infl* (depending on the tense-system in the language under study). We can define a very general meta-postulate for inclusion, based on the usual notion of *subsumption* (familiar from unification grammars). We call a *feature structure* a directed acyclic graph where features are arcs and values are vertices (and possibly all the sub-feature structure which results from all the arcs starting from that vertex). It is supposed that no two arcs are labelled by the same feature. A feature structure F1 is said to subsume a feature structure F2 if and only if all the features which are present in F1 are also present in F2, and with the same value. More precisely, we have:

Definition. F1 subsumes F2 (we write F1   F2) if and only if:

• for all (f, v)   F1, there exists (f, w)   F2, and if f is an atomic-valued feature, v = w, and if not, v subsumes w.

Metapostulate of inclusion.

• for every values α, β of either agr_N, agr_V or infl, we have:

[Incl] $\Diamond^*_\alpha A \rightarrow \Diamond^*_\beta A$ if and only if β   α

4.2. An example of agreement-inflection

Let us demonstrate the sequent:

$$(Marie, (lit, (souvent, Baudelaire))) \vdash \Box^0_{agr}\ \mathbf{vp}$$

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[6] If we still follow Chomsky, Infl is strong in languages with a rich verbal morphology (like it is the case in Romance languages).

[7] Except for the case of Verb-Object agreement in French, where the past participle must agree with the object also according to gender, but this case will not be dealt with in this paper (see Chomsky [9], pp146-150).

with the following assignments

Marie := $\Box_{agr(f,\,s,\,3)}\Box_k$**np**; *Baudelaire* := $\Box_{agr(m,\,s,\,3)}\Box_k$**np**;
lit ::= $\Box_{agrV(s,\,3)}\Box_{infl}$ **(vp/np)/np**; *souvent* := **(vp/np)/(vp/np)**

$$\dfrac{\dfrac{\dfrac{\dfrac{\ldots}{\mathbf{vp/np} \vdash \mathbf{vp/np}} \quad \dfrac{..[\delta]..}{(\Box_k\mathbf{np},\ \mathbf{vp/np}) \vdash \mathbf{vp}}}{(\Box_k\mathbf{np},\ (\mathbf{(vp/np)/(vp/np)},\ \mathbf{vp/np})) \vdash \mathbf{vp}}\,[/L]}{\mathbf{np} \vdash \mathbf{np} \qquad (\Box_k\mathbf{np},\ (souvent,\ \mathbf{vp/np})) \vdash \mathbf{vp}}\,[lex]}{(\Box_k\mathbf{np},\ (souvent,\ (\mathbf{(vp/np)/np},\ \mathbf{np}))) \vdash \mathbf{vp}}\,[/L]$$

$$[\text{———}\ [\Box L]$$

$$(\Box_k\mathbf{np},\ (souvent,\ (\mathbf{(vp/np)/np},\ (\Box_k\ \mathbf{np})^{kobj}))) \vdash \mathbf{vp}$$
$$\text{———}\ [K2]$$
$$(\Box_k\mathbf{np},\ (souvent,\ ((\mathbf{(vp/np)/np},\ \Box_k\ \mathbf{np}))^{kobj})) \vdash \mathbf{vp}$$
$$\text{———}\ [K2]$$
$$(\Box_k\mathbf{np},\ ((souvent,\ (\mathbf{(vp/np)/np},\ \Box_k\ \mathbf{np}))^{kobj}) \vdash \mathbf{vp}$$
$$\text{———}\ [K2]$$
$$(\Box_k\mathbf{np},\ ((souvent,\ (\mathbf{(vp/np)/np},\ \Box_k\ \mathbf{np})))^{kobj}) \vdash \mathbf{vp}$$
$$\text{———}\ [\Box L]$$
$$(\Box_k\mathbf{np},\ ((souvent,\ (\mathbf{(vp/np)/np},\ (\Box_{agr(m,\,s,\,3)}\ \Box_k\ \mathbf{np})^{\Diamond agrO})))^{kobj}) \vdash \mathbf{vp}$$
$$\text{———}\ [lex]$$
$$(\Box_k\mathbf{np},\ ((souvent,\ (\mathbf{(vp/np)/np},\ (Baudelaire)^{\Diamond agrO})))^{kobj}) \vdash \mathbf{vp}$$
$$\text{———}\ [K2]$$
$$(\Box_k\mathbf{np},\ ((souvent,\ ((\mathbf{(vp/np)/np}, Baudelaire\ ))^{\Diamond agrO}))^{kobj}) \vdash \mathbf{vp}$$
$$\text{———}\ [K2]$$
$$(\Box_k\mathbf{np},\ ((souvent,\ (\mathbf{(vp/np)/np}, Baudelaire\ )))^{\Diamond 0agrO}) \vdash \mathbf{vp}$$
$$\text{———}\ [\Box L]$$
$$(\Box_k\mathbf{np},\ ((souvent,\ ((\Box_{infl}\ \mathbf{(vp/np)/np})^{\Diamond infl}, Baudelaire\ )))^{\Diamond 0agrO}) \vdash \mathbf{vp}$$
$$\text{———}\ [K1]$$
$$(\Box_k\mathbf{np},\ ((souvent,\ ((\Box_{infl}\ \mathbf{(vp/np)/np}, Baudelaire\ ))^{\Diamond infl}))^{\Diamond 0agrO}) \vdash \mathbf{vp}$$
$$\text{———}\ [N_0W]$$
$$(\Box_k\mathbf{np},\ (((souvent,\ Baudelaire),\ \Box_{infl}\ \mathbf{(vp/np)/np}))^{\Diamond N0,infl}) \vdash \mathbf{vp}$$
$$\text{———}\ [S_0N_0]$$
$$(\Box_k\mathbf{np},\ ((\Box_{infl}\ \mathbf{(vp/np)/np},\ (souvent,\ Baudelaire)))^{\Diamond S0,infl}) \vdash \mathbf{vp}$$
$$\text{———}\ [\Box L]$$

$$(\Box_k \mathbf{np}, (((\Box_{agrV(s,3)} \Box_{infl} \, (\mathbf{vp/np})/\mathbf{np})^{\Diamond agrV(s,3)}, (souvent, Baudelaire)))^{\Diamond S0,infl}) \vdash \mathbf{vp}$$

[lex]

$$(\Box_k \mathbf{np}, (((lit)^{\Diamond agrV(s,3)}, (souvent, Baudelaire)))^{\Diamond S0,infl}) \vdash \mathbf{vp}$$

[Agr-Infl]

$$(\Box_k \mathbf{np}, ((lit, (souvent, Baudelaire)))^{\Diamond 0agrV(s,3)}) \vdash \mathbf{vp}$$

[Incl]

$$(\Box_k \mathbf{np}, ((lit, (souvent, Baudelaire)))^{\Diamond 0agrV(f,s,3)}) \vdash \mathbf{vp}$$

[$\Box$ L]

$$((\Box_{agr(f,s,3)} \Box_k \mathbf{np})^{\Diamond agrN}, ((lit, (souvent, Baudelaire)))^{\Diamond 0agrV}) \vdash \mathbf{vp}$$

[lex]

$$((Marie)^{\Diamond agrN(X)}, ((lit, (souvent, Baudelaire)))^{\Diamond 0agrV(X)}) \vdash \mathbf{vp}$$

[K_{agr}]

$$((Marie, (lit, (souvent, Baudelaire))))^{\Diamond 0agr(X)} \vdash \mathbf{vp}$$

[$\Box$R]

$$(Marie, (lit, (souvent, Baudelaire))) \vdash \Box^0_{agr(X)} \mathbf{vp}$$

Remark 1. We have slightly changed our conventions: we assume the modality $\Diamond^{agr(X)}$, where X is the variable which must be instanciated. In the same way, agr_N and agr_V are respectively transformed into $agr_N(X)$ and $agr_V(X)$.

Remark 2. [δ] succeeds if we modify [K_{agr}] in the following way:

[K_{agr}] $\Diamond^0_{agr}(A \bullet B) \rightarrow \Diamond^{S0}_{knom}(\Diamond_{agrN} A \bullet \Diamond^0_{agrV} B)$

in order that what remains to prove at the top of this deduction is in fact

$$((\Box_k \mathbf{np}, \mathbf{vp/np}))^{\Diamond S0knom} \vdash \mathbf{vp}$$

Let us give a few comments on this deduction. It shows that, in French, because the infl-mode is strong, in every sentence of this type, the verbal head has been moved, but when analyzing such a sentence, the original position is retrieved in such a way that the adverb (here *souvent*) gets its real scope.

5. Conclusion and discussion: reusability and economy

We think that many syntactic problems can be solved in a multimodal logical framework. The main contribution of such a framework to linguistics is *epistemological*. Logic is a precious tool for analyzing complex phenomena that sometimes have a very informal description, such as the Chomskyan theory of language. At this point, logic provides more and more refined concepts that allow a deep and precise analysis of the questions under study, such as moving constituents, agreement or inflection. For instance, Linear Logic provides a very good account of *resource-sensitivity*, a very widespread property in language. Of course, we cannot be satisfied with mere *substructural* logics (like the Lambek calculus) as there are many respects in which the limitation of resources is not so strict as the use of substructural logics would make us expect. For instance, agreement features are shared by two or more constituents in the sentence. This fact is expressed by the use of a modality which is subject to a *distributivity* postulate. Moreover, the simple fact that constituents can move is contradictory with the rigid frame provided by the Lambek calculus. Structural modalities allowing *permutations* are then needed, but also, as we tried to show, structural modalities *enforcing* permutations are needed. The display of all the concepts we need, such as hierarchically ordered modalities, the postulates to which they obey, the distinctions between strong, neutral and weak show us what is the exact price of adopting such and such a theory, and above all, compels the linguist to answer to some questions arising from the logical analysis. In this sense, linguistics could really become a scientific field (as scientific as physics which uses another part of mathematics in order to gain a deep understanding of nature, for instance).

Before ending, we point out some questions which arise from the above analysis. First, as we said above, the resource-sensitivity is not so strict as we could expect. There are many cases where some nominal phrase is reused, contrarily to the famous cancellation schemes of Categorial Grammar. If we have for instance (Pollock [30]):

(4) *Marie semble s'avérer idiote*

(Mary seems to turn out to be an idiot)

the **np** *Marie* is used three times as a subject, for each of the following positions (designated by an underline):

(4') Marie semble _ s'avérer _ idiote

The resource sensitivity must therefore be very relativized. This leads us to assume that, in fact, categories like **np** are given with a special modality which allows them to be contracted and perhaps weakened. This "special" modality is close to the exponential "**!**" of Linear Logic, but it seems to obey to the following particular contraction postulate (here expressed as a sequent rule)

$$[!\,C] \qquad \frac{\Gamma[(!\Delta_1, (\Delta_2, (!\Delta_1, \Delta_3)))] \vdash A}{\Gamma[(!\Delta_1, (\Delta_2, \Delta_3))] \vdash A}$$

Let us assume that a nominal phrase is assigned the type (here ignoring *agr*): $\Box_k$ **!np**. The essential point here is that *case* is used as a kind of *key*: when the NP is unlocked (i.e. its modality $\Box$ is removed) then it is usable as many times we require. This solution is a good compromise. There is still resource sensitivity (we can have neither **Marie Julie semble s'avérer idiote*, nor **semble s'avérer idiote*) but it is limited to the formal feature *case*, which is at the same time a key which allows the **np** *Marie* to be used several times *at different places*. If this analysis turns out to be correct (something we shall not try to demonstrate formally in the limits of this paper), it will lead us to the conclusion that, surprisingly, it is not in fact the Lambek calculus which occupies the center of the system, but modal logics. The resource-sensitivity would be, in effect, mainly located in the fact that when the [$\Box$L]-rule is used once, it can no longer be used (things being viewed from the bottom), thus guaranteeing that modalities are removed once and only once.

Another point to discuss in the future is the role devoted to so-called "Principles of Economy of Derivation" which are often mentioned in the Chomskyan literature. The point is that when an element is displaced for checking a formal feature, *it always stops at the nearest target* where the feature can be checked (Pollock [30]). We think that an account of such principles implies that we always restrict ourselves to *the shortest derivations* that we can obtain. This means that, as soon as a constituent can be cancelled, it stops and it will be cancelled at its current position.

In conclusion, we hope that we have presented enough convincing arguments in favour of the application of logics to other fields, such as linguistics. Of course, in doing so, we can think that logic is a little distorted. We are led to invent modalities (like our particular "**!**") which do not exactly match those that are used for purely logical purposes… but like nature *is not pure mathematics*, we cannot expect language *to be pure logic*!

We are indebted to other works realized in the multimodal categorial framework such as Kraak [15], Versmissen [34], Heylen [14].

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