Logica Trianguli, 1, 1997, 115-141

TARSKI ON LOGICAL ENTITIES*

Luis VILLEGAS-FORERO, Janusz MACIASZEK

Introduction

In this paper we will try to reconstruct Tarski's views on the *criterion of logicality* and the nature of *logical entities* in his 1966 lecture ([31]), in terms of an explicit ontological scheme of functional types, sequential frames and categories, which results from a simple adaptation of Montague's ([21]). We also sketch a historical background of investigations on logical constants and present various possible techniques applied.

There is at least one non trivial justification for the considerations presented. Even though Tarski was not the first to formulate the permutation invariance as ***the*** criterion of logicality[1], he was probably the first to apply it ontologically to objects of various types (in a first-order framework). Nevertheless, this Tarski's contribution is hardly mentioned in the bibliography of logical constants. We intend to fill this gap up to some extent.

With regard to the treatment of Tarski's actual views, our approach is, in a sense, similar to Simon's ([28]). We believe, however, that our account is less philosophical though more systematic and general. There are of course other approaches similar to ours, but they use different ontological backgrounds (see e.g. van Benthem [34] and Westerstahl [38]).

As Simon [28] has pointed out, Tarski faces the logicality problem from a strictly ontological, language independent, point of view:

---

* This work was supported by the Spanish Ministry of Education and Science under Project PB95-0125-C06-02 ("La función de los factores semánticos en la evaluación del conocimiento científico") and by the Xunta de Galicia (Counsellery of Education and Universities) under Project XUGA20509B96 ("Ontosemantica de los nombres comunes: términos científicos y géneros (naturales y no-naturales)").

1 vid. further down, section II.1, at the beginning.

> "... I shall not discuss the general question 'what is logic?' I take logic to be a science, a system of true sentences, and the sentences contain terms denoting certain notions, logical notions. I shall be concerned here with only one aspect of problem, the problem of logical notions, but not for instance with the problem of logical truths." ([31: 145).

Thus, in Tarski's account issues such as the semantic nature of the logical expressions, *qua* expressions, truth, and logical consequence do not arise. This is not in fact the usual approach. Many papers on logical constants do not distinguish between being a denotation of a logical constant and being a logical entity, or at least this distinction is not clearly expressed. This point may appear trivial at first sight, but it is nonetheless important, since some recent critiques[2], or revisions of Tarskian account of logic and semantics perhaps have not taken into consideration two different levels in Tarski's work: (i) A *primary* or basic logicality that pertains to logical notions and/or entities, *qua ontic citizens*. (ii) A *secondary* logicality, which is *semantically induced* in certain expressions or full categories of expressions of an object-language by means of different kinds of *admissibility constraints* upon their possible interpretations. The relations between these two levels are intricate and difficult to elucidate. We would like to concentrate here on the first perspective[3].

Before starting the reconstruction of Tarski's view it is necessary to make the followings remarks:

(1) The ontology required by Tarski is purely formal. He does not seem to worry about the substantive nature of logical entities, i.e. whether they are physical, mental or whether they belong to a platonic, fregean, bolzanian or popperian third realm.

(2) Without expressing agreement or disagreement with Tarski's ontological criterion of logicality - invariance under arbitrary permutations of the universe - logical entities seem to be unavoidably bound - or perhaps to be represented by - typed functional constructs from the individuals of the universe of discourse and extensionally associated with set-theoretical constructs. Thus, although it is controversial whether Tarski did support, in general, a conceptual, disciplinary or epistemic reduction of logic to mathematics, through set theory, it seems plausible to interpret Tarski's 1966 paper as supporting the view that logical enti-

---

[2] Cf. by instance, Etchemendy ([9], [10]).

[3] We presented a view on the second approach in [17], and, above all, in [36]. A symbiotic treatment of both levels can be found in Sher ([27]).

ties are ontologically reducible to - or can be ontologically represented by - mathematical entities by means of a typed set theory.

(3) In order to clear a possible ambiguity in Tarski it is necessary to make a distinction he does not provide: One thing is a *logical notion*, which, in the present non-epistemic sense, can be understood as a *fregean concept* or perhaps a *suppesian set-theoretical property.* Another thing is a *logical entity* which is something that can fall under a logical notion. In this sense, it is also convenient to distinguish between the fixed domain (universe) and the non-fixed domain account. If you consider one single universe, logical notions and logical entities simply collapse. To be precise, for each logical notion there exists a unique logical entity falling under this notion. However, if changes of the universe of discourse are taken into account, the situation also changes. Namely, for (almost) every logical notion there exist different logical entities which fall under the notion.

Thus, following Simon's views - attributed by him to John Corcoran - about *ramification* of logical entities in different universes, we could now think that *logical notions* can be represented by trans-typed functions from universes (or, in general, frames) to logical entities. These functions should select relatively to each frame, the logical entity that falls under the given logical notion. We will try to provide ahead, with more precise explanations about these points.

The paper consists of two sections. The first concentrates on explicit reconstruction of Tarski's opinions. In the second, we present historical and contextual sketch of the problem of logical constants and the criteria of logicality in the analytical tradition.

I. RECONSTRUCTION OF TARSKI'S PROPOSAL

1. Underlying formal ontology

We first define an ontological scheme which intends to reconstruct and generalise a certain formal ontology based on Tarskian semantics. We will present this scheme in terms of an abstract type structure of objects (individuals and truth-values) and functions, similar to the one of Montague ([21]) and of a frame we call a "*first-order Tarski frame*". The combination of both systems generates the categorised collection of the admissible entities that can be extra logical or logical. We call this ontology "quasi-intensional", as the sequences of the objects of the universe that appear in it, play the role of indexes or points of reference[4]. We will represent this notion by introducing type **q** (syncategorematically) into the typology, in order to define types and categories of **q**-functions called from now on "*quasi-intensions*", or simply *q-intensions*.

We define *Tarski ontological system* O as a sextuple <T, F, $(K_{\tau,F})_{\tau}$ T, $(F_i)_i$ I, $(R_j)_j$ J, <**C**, •>>, where T is an abstract *Typology* of entities, F is a *First Order Tarski Frame*, $(K_{\tau,F})_{\tau}$ T is a family of *Categories* of entities, typified by T and generated by F, < $(K_{\tau,F})_{\tau}$ T, $(F_i)_i$ I> is an *Algebra* of ontological operations , $(R_j)_j$ J is a family of *type-change Rules*, and <**C**, •> is a *replacement system*.

1.1. Typology

Let the *Typology* T, the set of types of entities, be the smallest set such that

1. **e, t** T, where **e** is the type of *individuals* and **t**, the type of *truth values*.
2. If σ, τ T, then <σ**,** τ> T, where <σ**,** τ> is the type of *functions* from entities of type σ to entities of type τ.
3. If τ T then <**q,** τ> T, where **q** is the syncategorematic type of infinite *sequences* of individuals[5] and <**q,** τ> is the type of functions from infinite sequences to things of type τ.

The types can be divided into *extensional and q-intensional* ones. T_{ext}, the set of *extensional types,* is the smallest subset of T such that: (1) **e**, **t** T_{ext}. (2) if σ ,τ T_{ext}, then <σ**,** τ> T_{ext}. The set, T_q, of *q-intensional types* is therefore T ¾T $_{ext}$, and T_{pq}, the set of types of *proper q-intensions,* is the smallest subset of T_q such that if σ T, then <**q,** σ> T_{pq}. In any case, <**q,** σ> is the type of *q-intensions* corresponding to type τ entities. We can define as *basic types* of *q-intensions* the type <**q, t**> (*q-propositions*, and the type <**q, e**> (*q-individual concepts*).

Among the types defined we can distinguish between the so-called *relational and the q-relational types.* T_{rel}, the set of *relational* types is

---

[4] They play an analogous role to *possible worlds* of the modal ontology from in Carnapian tradition.

[5] **q** will play a role analogous to that of Montague's [21] **s**, which corresponds to possible worlds.

the smallest subset of T such that: (1) if σ T, then <σ, **t**> T_{rel}. (2) if σ T and τ T_{rel}, then <σ, τ> T_{rel}, where <σ, **t**> is the type of *monary relations* or *properties* of type σ entities. T_{qrel}, the set of *q-relational* types is the smallest subset of T_{pq} such that if τ T_{rel}, then <**q**, τ> T_{qrel}.

In particular, the set T_{tff} of types of *truth-functions* is the smallest subset of T_{rel} such that: (1) <**t, t**> T_{tff}. (2) if σ T_{tff}, then <**t**, σ> T_{tff}. And, the set T_{tFF}, of types of *truth-functionals* is the smallest subset of T_{rel} such that: (1) if σ T_{tff}, then <σ, **t**> T_{tFF}. (2) if σ, τ T_{tff} T_{tFF}, then <σ, τ > T_{tFF}.

1.2. First-order Tarski frames

The ontology presented is developed in terms of the notion of *First-Order Tarski Frame*. The frame furnishes with the *material* - or better, *real* - *content* of the ontological system, because the type structure is compatible with changes in the components of the frames.

A *First-Order Tarski Frame* (*FTF*) *F* is defined as an ordered triple <E, SEQ, **acc**> such that,

1. E is a denumerable not empty set of individuals, the *universe* of *F*.

2. SEQ = E^{ω} is the set of infinite sequences of members of E, considered as the set of *indexes* of *F*.

3. **acc**, the *accessibility* function, is a function from SEQ × $\mathbf{Z}^+$ to $\{\mathbf{F}, \mathbf{T}\}^{SEQ}$ such that for all **s**, **s**' SEQ and **i** $\mathbf{Z}^+$, **acc**(**s**, **i**)(**s**') = **T** iff $pr_{\mathbf{j}}$(**s**') = $pr_{\mathbf{j}}$(**s**) for all **j** $\mathbf{Z}^+$ other than **i**; and **acc**(**s**, **i**)(**s**') = **F** otherwise, where, as usual, **T** represents the value *true* and **F**, the value *false* [6].

1.3. Ontological categories

Now let us relate *entity types* with the *entities typified*. Given a type system T and a *FTF* *F*, we define the *family of categories* of T,*F*-

---

[6] Thus, the values of the accessibility function are type <**q, t**> q-intensions i.e., *q-propositions*, and the notion of accessibility incorporates the relation between sequences which in standard Tarski semantics serves to define satisfaction to quantified formulae.

entities, $(K_{\tau,F})_{\tau \quad T}$, as the smallest family such that

1. $K_{\mathbf{e},\,F} = E$
2. $K_{\mathbf{t},\,F} = \{\mathsf{F}, \mathsf{T}\}$[7]
3. For all $\sigma, \tau \quad T$, $K_{<\sigma,\,\tau>,F} = K_{\tau,\,F}{}^{K_{\sigma,\,F}}$
4. For all $\sigma \quad T$, $K_{<\mathbf{q},\,\sigma>,\,F} = K_{\sigma,\,F}{}^{SEQ}$[8]

Let $(K_{\sigma,F})_{\tau \quad T})$, or briefly ENT_O, be the collection of *entities* T, F*-admissible* in the ontology. $K_{\mathbf{e},\,F} \quad K_{\mathbf{t},\,F}$ is the collection OB_O of *objects* of the system. $FUNC_O = ENT_O — OB_O$ is the collection of *functions* of the system. Let the relation from entities to the types of entities be such that θ **:** σ ("θ is of type σ") iff. $\theta \quad K_{\sigma,\,F}$.

1.4. Other ontological notions

The next ontological notions that should be defined are *ontological operations***,** *rules* and *conditions upon the rules***.**

(1) $<ENT_O, (F_i)_{i \quad I}>$ is the *ontological algebra of* O, whose operations are general total modes of ontological articulation.
(2) $(R_j)_{j \quad J}$ is a family of rules which limit uncorrect use of the operations. A given operation is usually limited by several rules. Each rule has: i) a *type-change condition*, which connects certain arguments-types with a determinate value-type and may be founded upon an analogous rule of conditional logic, in the way studied by van Benthem ([34]), and ii) a *change procedure* serving to choose specific values for arguments in the appropriate types. Thus, for each $j \quad J$, R_j is an ordered triple F_i,

---

[7] Tarski identified the value *True* with the *universe* E, and the value *False*, with . Following Lindstrom ([16]), *True* and *False* are represented as { }, , respectively (they are the only 0-place relations on E). The choice is not as harmless as it appears, because, if changes of universe are permitted, they shall bring on a domino effect upon the trans-frame invariances of logical entities which constructively depend on the value *True*.

[8] In particular, $K_{<\mathbf{q,e}>,F}$ is the category of *q-individual concepts*, and $K_{<\mathbf{q,t}>,F}$ is the category of *q-propositions*. Note that q-propositions are simply the characteristic functions of sets of sequences defined as interpretation of formulae by McCarthy [19] and Montague [20].

$\sigma_1, \ldots, \sigma_k/\tau$, $¶_j$ where F_i is the k-ary limited operation, $\sigma_1, \ldots, \sigma_k$, τ T, $\sigma_1, \ldots, \sigma_k/\tau$ is the relevant type-change condition, and $¶_j$ is the change procedure. The ontological rule "says" that if $\theta_1, \ldots, \theta_k$ ENT_O and $\theta_1 : \sigma_1, \ldots, \theta_k : \sigma_k$, then (1) $F_i(\theta_1, \ldots, \theta_k) : \tau$. (2) if $\theta : \tau$, then $F_i(\theta_1, \ldots, \theta_k) = \theta$ iff $¶_j(\theta_1, \ldots, \theta_k, \theta)$.

Example

$R_G =$ F_i, $<\tau, \tau'> / <<\sigma, \tau>, <\sigma, \tau'>>$, $¶_G$ is the general form of the rule, whose type-change condition is usually know as *Geach's condition*, and its change procedure is:

if $\mathbf{f} : <\tau, \tau'>$ and $\mathbf{g} : <<\sigma, \tau>, <\sigma, \tau'>>$, then $F_i(\mathbf{f}) = \mathbf{g}$ iff $¶_G(\mathbf{f}, \mathbf{g})$ iff for all $\mathbf{h} : <\sigma, \tau>$ and all $\theta : \sigma$, $\mathbf{g}(\mathbf{h})(\theta) = \mathbf{f}(\mathbf{h}(\theta))$[9].

1.5. Replacement system

We consider that the entities generated by the typology and the frame are structured entities, whose components may be systematically replaced by other entities, bringing forth, as a result, new entities. Thus it is possible to construct a *replacement system*, $<\mathbf{C}, \bullet>$, in the sense of Aczel [1], with a function of assignation of *constituents*, $\mathbf{C}$, an a *replacement operation*, $\bullet$.

1.5.1. Constituency function

$\mathbf{C}$ is a non injective function which to each entity θ ENT_O assigns the set $\mathbf{C}(\theta)$ of its *constituents* in the following way:

Constituents of objects

i) if $\mathbf{a} : \mathbf{e}$, then $\mathbf{C}(\mathbf{a})=$.

ii) if $\mathbf{v} : \mathbf{t}$, then $\mathbf{C}(\mathbf{v}) = \mathbf{v}$.

Constituents of functions which are not q-intensions

If $\mathbf{f} : <\sigma, \tau>$, then $\mathbf{C}(\mathbf{f})$ can be inductively constructed over the complexity of $\mathbf{f}$ in the following way :

Let $\mathbf{IC}(\mathbf{f}) = K_{\sigma, F}$ $\mathrm{Rang}(\mathbf{f})$, be the set of *immediate constituents* of $\mathbf{f}$.

1. $\mathbf{OBC}^1(\mathbf{f}) = \mathbf{IC}(\mathbf{f}) \cap OB_O$, is the set of *level 1 objectual*

---

[9] $R_{IG} =$ F_i, $<\tau, \tau'> / <<\mathbf{q}, \tau>, <\mathbf{q}, \tau'>>$, $¶_{IG}$ is the q-intensional version of R_G, with a similar change procedure.

constituents of **f**.

$\mathbf{FC}^1(\mathbf{f}) = \mathbf{IC}(\mathbf{f})$ — $\mathbf{OBC}^1(\mathbf{f})$, is the set of *level 1 functional constituents* of **f**.

2. $\mathbf{OBC}^2(\mathbf{f}) = (\ _{\mathbf{g_i}\ \ \mathbf{FC}^1(\mathbf{f})}\ \mathbf{IC}(\mathbf{g_i})) \cap OB_O$ is the set of *level 2 objectual constituents* of **f**.

$\mathbf{FC}^2(\mathbf{f}) = (\ _{\mathbf{g_i}\ \ \mathbf{FC}^1(\mathbf{f})}\ \mathbf{IC}(\mathbf{g_i}))$ — $\mathbf{OBC}^2(\mathbf{f})$ is the set of *level 2 functional constituents* of **f**.

3. $\mathbf{OBC}^3(f) = (\ _{\mathbf{g_j}\ \ \mathbf{FC}^2(\mathbf{f})}\ \mathbf{IC}(\mathbf{g_j})) \cap OB_O$ is the set of *level 3 objectual constituents* of **f**.

$\mathbf{FC}^3(f) = (\ _{\mathbf{g_j}\ \ \mathbf{FC}^2(\mathbf{f})}\ \mathbf{IC}(\mathbf{g_j}))$ — $\mathbf{OBC}^3(\mathbf{f})$ is the set of *level 3 functional constituents* of **f**.

m = Min {k **N**½**OBC**^{k}(**f**)= & **FC**^{k}(**f**)= }.

We define now:

$\mathbf{OBC}(\mathbf{f}) = \ _{i=1}^{m-1}\ \mathbf{OBC}^i(\mathbf{f})$, as the set of *basic constituents* of **f**.

$\mathbf{OBC}_\mathbf{e}(\mathbf{f}) = \mathbf{OBC}(\mathbf{f}) \cap K_\mathbf{e}$, *F*, as the set of *individual constituents* of **f**.

$\mathbf{OBC}_\mathbf{t}(f) = \mathbf{OBC}(\mathbf{f}) \cap K_\mathbf{t}$, *F*, as the set of *truth-value constituents* of **f**.

$\mathbf{FC}(\mathbf{f}) = \ _{i=1}^{m-1}\ \mathbf{FC}^i(\mathbf{f})$, as the set of *functional constituents* of **f**.

$\mathbf{C}(\mathbf{f}) = \mathbf{OBC}(\mathbf{f})\ \ \ \mathbf{FC}(\mathbf{f})$, as the set of *all constituents* of **f**.

Constituents of q-intensions.

If **f: <q, τ>,** then

$\mathbf{C}(\mathbf{f}) = \ _{i=1}^{\omega}\ \{\mathbf{pr_i}(\mathbf{s})½\mathbf{s}\ \ \mathrm{SEQ}\}\ \ \mathrm{Rang}(\mathbf{f})\ \ \ (\ _{\theta\ \ \mathrm{Rang}(\mathbf{f})}\ \mathbf{C}(\theta))$.

Obviously, $\bigcup_{i=1}^{\omega} \{\mathbf{pr_i}(\mathbf{s}) | \mathbf{s} \in \text{SEQ}\} = \text{E}$.

Proposition. If θ y θ' ∈ ENT_O , then $\theta \in \mathbf{C}(\theta') \Rightarrow \mathbf{C}(\theta) \subseteq \mathbf{C}(\theta')$.

1.5.2. Replacement operation[10]

•: $ENT_O \times \{\mathbf{r} : \langle \mathbf{e}, \mathbf{e} \rangle | \mathbf{r} \in \text{BIJEC}\}$ is an operation such that:

1. if $\mathbf{a} : \mathbf{e}$, then $\mathbf{a}\bullet\mathbf{r} = \mathbf{r}(\mathbf{a})$.
2. if $\mathbf{v} : \mathbf{t}$, then $\mathbf{v}\bullet\mathbf{r} = \mathbf{v}$.
3. if $\mathbf{f} : \langle\sigma_1, \langle\sigma_2, \ldots, \langle\sigma_\mathbf{n}, \tau\rangle\ldots\rangle$ is a n-ary (n ≥1) function and σ_1, $\sigma_2, \ldots, \sigma_\mathbf{n}$, τ are non necessarily different types, then for all $\theta_1 : \sigma_1$, $\theta_2 : \sigma_2, \ldots, \theta_\mathbf{n} : \sigma_\mathbf{n}$, $\mathbf{f}(\theta_1)(\theta_2)\ldots(\theta_\mathbf{n}) = \mathbf{f}\bullet\mathbf{r}(\theta_1\bullet\mathbf{r})(\theta_2\bullet\mathbf{r})\ldots(\theta_\mathbf{n}\bullet\mathbf{r})$.

Thus • is a *replacement operation* such that, for each *bijection* of the universe E onto itself, θ•**r** is the structured entity of the same type that θ , which results of replacing *down-top* each component, θ', of θ by its value θ'•**r**, in such a way that $\mathbf{C}(\theta\bullet r) = \{\theta'\bullet\mathbf{r} | \theta' \in \mathbf{C}(\theta)\}$.

2. Tarski-like logical entities

2.1. Logicality criterion

> “In the extreme case, we would consider the class of *all* one-one transformations of the space, or universe of discourse, or “world”, onto itself. What will be the science which deals with the notions invariant under this widest class of transformations? Here we will have very few notions, all of a very general character. I suggest that they are the logical notions, that we call a notion 'logical' if it is invariant under all possible one-one transformation of the world onto itself” (Tarski [31]: 149).

[10] This operation does correspond to the method, informally characterised by Tarski ([29]: 152), of *chain* transformations, starting with “transformations of the 'world' onto itself”.

θ is a *Tarski logical entity* in the ontological system O iff. for all bijection **r: <e, e>**, θ•**r** = θ.

2.2. Categories of logical entities

$(K_{log,\ \tau,F})_{\tau}$ T, is the family of categories of logical entities such that for all τ T, $K_{log,\ \tau,\ F}$, is the set of logical entities of this type, relatively to the frame F. This set is to be determined relatively to each frame. As we said in the introduction, if changes of frames or universes are permitted, each *logical notion* is to be *the same*, but the *specific entity* which falls under the notion *can vary*. So, the abstract basis for building each logical category is the same in all possible frames, but its inhabitants may be different. Moreover, if a quasi-intensional ontology is given, then also quasi-intensional logical entities appear, which were not supposed directly by Tarski. First we list some categories of logical entities in Tarski's paper:

1. *Individuals*

> "... we can start with individuals, with objects of the lowest type,... There are no logical notions of this type, simply because we can always find a transformation of the world onto itself where one individual is transformed into a different individual." (Tarski [31]: 150): $K_{log,\mathbf{e}},\ F =$.

2. *Truth-value based entities*

> "If we proceed to the next level, to classes of individuals,..., there are exactly two classes of individuals which are logical: the universal class and the empty class. Only these two classes are invariant under every transformation of the universe onto itself." (Tarski [31]: 150)[11].

[11] The editor of Tarski's paper, J. Corcoran, remarks on page 150, note 6: "In his Buffalo lecture, Tarski indicated that the present remarks apply to 'notions' taken in the narrow sense of sets, classes of sets, etc., but that the truth-functions, quantifiers, relation-operators, etc. of *Principia Mathematica* can be construed as notions in the narrow sense and, so construed, the present remarks apply equally to them. For example, construing the truth-values T and F as the universe of discourse and the null set leads immediately to construing truth-functions as (higher-order) notions". (Tarski [11]: 150, note 6. from J. Corcoran (ed.)).

Let T_{tv}, the set of types of *truth-value based entities*, be the smallest set such that: (1) **t** T_{tv}. (2) If σ, τ T_{tv}, then <σ**,** τ> T_{tv}[12]. Then, for all **t** T_{tv}, $K_{log,\mathbf{t}, F} = K_{\mathbf{t}, F}$.

3. *Relations between individuals and, in general, between entities of the same type*

> "If we... consider binary relations, a simple argument shows that there are only four binary relations which are logical in this sense: the universal relation..., the empty relation..., the identity relation..., and its opposite, the diversity relation... If you consider ternary relations, quaternary relations, and so on, the situation is similar: for each of these you will have a small finite number of logical relations." (Tarski [31]: 150).

f $K_{log, <\sigma, <\sigma,...,<\sigma, \mathbf{t}>...>, F}$ if **f** is a n-ary (n≥2) relation such that:

i) for each $\theta_1, \theta_2, \ldots, \theta_n$**:** σ, **f**$(\theta_1)(\theta_2)\ldots(\theta_n)$ = **T** iff $\theta_1 = \theta_2 = \ldots = \theta_n$ (*n-ary Identity*) or

ii) for each $\theta_1, \theta_2, \ldots, \theta_n$**:** σ, **f**$(\theta_1)(\theta_2)\ldots(\theta_n)$ = **T** iff for each **i**, **j**, $\theta_i \neq \theta_j$ (*n-ary Difference*) or

iii) for each $\theta_1, \theta_2, \ldots, \theta_n$**:** σ, **f**$(\theta_1)(\theta_2)\ldots(\theta_n)$ =**T** (*n-ary universal relation*) or

iv) for each $\theta_1, \theta_2, \ldots, \theta_n$**:** σ, **f**$(\theta_1)(\theta_2)\ldots(\theta_n)$ = **F**(*n-ary empty relation*).

4. *Cardinality properties*

> "It turns out that the only properties of classes (of individuals) that are logical are properties concerning the number of elements in these classes. That a class consists of three elements, or four elements... that is finite , or infinite - these are logical notions and are essentially the only logical notions on this level." (Tarski [31]: 151).

---

[12] Obviously, $T_{tv} = \{\mathbf{t}\}$ T_{tff} T_{tFF} where T_{tff}, T_{tFF} are, as above, the set of types of *truth-functions* and *truth-functionals*, respectively.

f $K_{log,<<\sigma, t>, t>, F}$, if for each **g:** <σ, **t**>, **f**(**g**) = **T** iff. there are exactly $\theta_1, \theta_2, ..., \theta_n$ **:** σ (1 ≤n≤½$K_{\sigma, F}$½) such that **g**(θ_1) = **g**(θ_2)= .... = **g**(θ_n) = **T**.

5. *Binary relations between n-ary relations defined in each type*

> "If you turn to... relations between classes, then the variety of logical notions increases. Here for the first time you come across many important and interesting logical relations, well known... I mean such things as inclusion between classes, disjointness of two classes, overlapping of two classes, and many others." (Tarski [31]: 151).

f $K_{log, <<\sigma, <\sigma,...,<\sigma, t>...>, <<\sigma, <\sigma,...,<\sigma, t>...>, t>>, F}$ if **f** is a binary relation between n-ary (n≥2) relations on $K_{\sigma,F}$ such that:

i) for each **g**, **h :** <σ, <σ**,...,**<σ**, t>...**>, **f**(**g**)(**h**) = **T** iff for each $\theta_1, \theta_2,...,\theta_n$**:** σ, if **g**($\theta_1$)($\theta_2$)...($\theta_n$) = **T**, then **h**($\theta_1$)($\theta_2$)...($\theta_n$) = **T** (*Inclusion*) or
ii) for each **g**, **h :** <σ, <σ**,...,**<σ**, t>...**>, **f**(**g**)(**h**) = **T** iff for each $\theta_1, \theta_2,...,\theta_n$**:** σ, if **g**($\theta_1$)($\theta_2$)...($\theta_n$) = **T**, then **h**($\theta_1$)($\theta_2$)...($\theta_n$) = **F** (*Disjointness*) or
iii) for each **g**, **h :** <σ, <σ**,...,**<σ**, t>...**>, **f**(**g**)(**h**) = **T** iff there is $\theta_1, \theta_2,...,\theta_n$**:** σ such that **g**($\theta_1$)($\theta_2$)...($\theta_n$) = **h**($\theta_1$)($\theta_2$)...($\theta_n$) = **T** (*Overlapping*) or
iv) for each **g**, **h :** <σ, <σ**,...,**<σ**, t>...**>, **f**(**g**)(**h**) = **T** iff there is $\theta_1, \theta_2,...,\theta_n$**:** σ such that **g**($\theta_1$)($\theta_2$)...($\theta_n$) = **T** and **h**($\theta_1$)($\theta_2$)...($\theta_n$) = **F** (*Not-inclusion*).

In particular, if **f:** <<σ**, t>, <**<σ**, t>, t**>>>, i.e. if it is a relation between properties of entities of type σ, the above conditions apply to the *four classical quantifications* over entities of type σ, *restricted* by a determined property of entities of this type.

6. *Membership relations and in general, identical functions in each type* [13]

> "Using this method [the method of *Principia Mathematica*] it is clear that membership relation is certainly a logical notion. It occurs in several types , for individuals are elements of classes of individuals, classes of individuals are elements of classes of classes of individuals, and so on. And by the very definition of an induced transformation it is invariant under every transformation of the world onto itself." (Tarski [29]: 153).

$\mathbf{f}$ $K_{log,<\sigma,\sigma>}$, *F* if for each $\theta : \sigma$, $\mathbf{f}(\theta) = \theta$.

Now let us introduce some categories of logical entities not mentioned by Tarski. We begin with extensional entities.

1. *Generalised complementation*

$\mathbf{f}$ $K_{log,<<\sigma_1,<\sigma_2, \ldots,<\sigma_n, \mathbf{t}>\ldots>, <\sigma_1,<\sigma_2, \ldots,<\sigma_n, \mathbf{t}>\ldots>>>}$, *F*, if $\sigma_1, \sigma_2, \ldots, \sigma_{\mathbf{n}}$ T ($n\geq 1$) are non-necessarily different types and for all $\mathbf{g}: <\sigma_1,<\sigma_2, \ldots,<\sigma_{\mathbf{n}}, \mathbf{t}>\ldots>$, $\theta_{\mathbf{1}}: \sigma_1$, $\theta_{\mathbf{2}}: \sigma_2$,.., and $\theta_{\mathbf{n}}: \sigma_{\mathbf{n}}$, $\mathbf{f}(\mathbf{g})(\theta_{\mathbf{1}})(\theta_{\mathbf{2}})\ldots(\theta_{\mathbf{n}}) = \mathsf{T}$ iff $g(\theta_{\mathbf{1}})(\theta_{\mathbf{2}})\ldots(\theta_{\mathbf{n}}))= \mathsf{F}$.

2. *Unrestricted classical quantifications over entities of any type*

$\mathbf{f}$ $K_{log,<<\sigma, \mathbf{t}>, \mathbf{t}>}$, *F* if:

i) for each $\mathbf{g}: <\sigma, \mathbf{t}>$, $\mathbf{f}(\mathbf{g}) = \mathsf{T}$ iff for each $\theta : \sigma$, $\mathbf{g}(\theta)= \mathsf{T}$ (*Universal Affirmative*) or

ii) for each $\mathbf{g}: <\sigma, \mathbf{t}>$, $\mathbf{f}(\mathbf{g}) = \mathsf{T}$ iff for some $\theta : \sigma$, $\mathbf{g}(\theta)= \mathsf{F}$ (*Particular Negative*) or

iii) for each $\mathbf{g}: <\sigma, \mathbf{t}>$, $\mathbf{f}(\mathbf{g}) = \mathsf{T}$ iff for each $\theta : \sigma$, $\mathbf{g}(\theta)= \mathsf{F}$ (*Universal Negative*) or

iv) for each $\mathbf{g}: <\sigma, \mathbf{t}>$, $\mathbf{f}(\mathbf{g}) = \mathsf{T}$ iff for some $\theta : \sigma$, $\mathbf{g}(\theta)= \mathsf{T}$ (*Particular Affirmative*).

---

[13] It is evident that for each type σ, the identity function $\mathbf{f}: <<\sigma,\mathbf{t}>,<\sigma,\mathbf{t}>>$, as the representation of the binary relation between entities of type σ and functions of type $<\sigma, \mathbf{t}>$, is the functional analogue to the membership relation.

3. *Restricted numerical quantifications over entities of any type*

f $K_{log,<<\sigma, t>, <<\sigma, t>, t>>>}$, *F*, if for each **g**, **h** : $<\sigma, t>$, **f**(**g**)(**h**) = **T** iff there are exactly $\theta_1, \theta_2, ..., \theta_n : \sigma$ (1 ≤ n ≤½K_σ, *F*½) such that **h**(θ_1) = **g**(θ_1) = **h**(θ_2) = **g**(θ_2)= .... = **h**(θ_n)= **g**(θ_n) = **T**.

Now let us turn to some *quasi-intensional* entities, that can be introduced in the ontology presented in this paper.

1. *Projective q-individual concepts*

f $K_{log,}$ <**q, e**>, if for each **s** SEQ, **f**(**s**) is the individual which occupies the **i**-th position in **s**, and so **f** is the **i**-th *projection* over SEQ or the **i**-th *projective q-individual concept*. In what follows such a **f** will be denoted by $\mathbf{pr_i}$.

2. *Some unary first-order q-intensional quantifications*

Let **f: <q, <<q, e>, <<q, t>, t>>>** be a q-relation between q-propositions and q-individual concepts, let $\mathbf{Acc_{s,i}}$ be the set characterized by the q-proposition of accessibility to sequence **s** in its **i**-th position, and let $\mathbf{T}_\pi$ ($\mathbf{F}_\pi$)} be, in general, the *truth (falsity)-set* of the q-proposition, π. So, **f** $K_{log,<q, <<q, e>, <<q, t>, t>>>}$, *F* if for each *projection index* **i**, *q-proposition* π, and sequence **s**:

1) **f**(**s**)($\mathbf{pr_i}$)(π) = **T** iff $\mathbf{Acc_{s,i}} \cap \mathbf{F}_\pi =$ (*Classical affirmative universal*) or

2) **f**(**s**)($\mathbf{pr_i}$)(π) = **T** iff $\mathbf{Acc_{s,i}} \cap \mathbf{F}_\pi \neq$ (*Classical negative particular*) or

3) **f**(**s**)($\mathbf{pr_i}$)(π) = **T** iff $\mathbf{Acc_{s,i}} \cap \mathbf{T}_\pi \neq$ (*Classical affirmative particular*) or

4) **f**(**s**)($\mathbf{pr_i}$)(π) = **T** iff $\mathbf{Acc_{s,i}} \cap \mathbf{T}_\pi =$ (*Classical negative universal*) or

5) **f**(**s**)($\mathbf{pr_i}$)(π) = **T** iff ½$\mathbf{Acc_{s,i}} \cap \mathbf{T}_\pi$½ = **n** (**n** ≥1) (*n-th exact affirmative*) or

6) **f**(**s**)($\mathbf{pr_i}$)(π) = **T** iff ½$\mathbf{Acc_{s,i}} \cap \mathbf{F}_\pi$½ = **n** (**n** ≥1) (*n-th exact negative*) or

7) **f**(**s**)($\mathbf{pr_i}$)(π) = **T** iff ½$\mathbf{Acc_{s,i}} \cap \mathbf{T}_\pi$½ > ½$\mathbf{Acc_{s,i}} \cap \mathbf{F}_\pi$½ (*Approximate superlative*) or

8) **f**(**s**)($\mathbf{pr_i}$)(π) = **T** iff ½$\mathbf{Acc_{s,i}} \cap \mathbf{T}_\pi$½ < ½$\mathbf{Acc_{s,i}} \cap \mathbf{F}_\pi$½ (*Approximate diminutive*) or

9) **f**(**s**)($\mathbf{pr_i}$)(π) = **T** (*Assertive neutral*) or

10) **f**(**s**)$\mathbf{pr_i}$)(π) = **F** (*Refutative neutral*) or

11) **f**(**s**)$\mathbf{pr_i}$)(π) = **T** iff π(**s**) = **T** (*Factual neutral*) or

12) $\mathbf{f}(\mathbf{s})\mathbf{pr_i})(\pi) = \mathsf{T}$ iff $\pi(\mathbf{s}) = \mathsf{F}$ (*Counterfactual neutral*)[14]

Below, we sketch some recursive procedures for constructing new logical entities from other logical entities.

1. *Constancy range*

$\mathbf{f}$ $K_{log,\ <\sigma,\ \tau>,\ F}$, if σ T^*, τ T, and $\mathbf{f}$ is a function such that it takes constant values in the set $K_{log,\ \tau,\ F}$, where $\mathsf{T}^* = \mathsf{T}$ $\{\mathbf{q}\}$.

2. *Persistence under type-change*

If $\mathbf{f}$ $K_{log,\sigma,\ F}$, and there is an ontological rule, R_j, such that $R_j =$ F_i, σ / τ ¶$_j$, then $F_i(\mathbf{f})$ $K_{log,\tau,\ F}$.

Thus, the collection LOG_O of logical entities of the ontological system O shall therefore be $(K_{log,\ \tau,\ F})_{\tau}$ T.

2.3. Logical indiscernibility

> "We cannot logically distinguish two classes from each other if each of them has exactly two individuals, because if you have two classes, each of which consists of two individuals, you can always find a transformation of the universe under which one of these classes is transformed into the other. Every logical property which belongs to one class of two individuals belongs to every class containing exactly two individuals." (Tarski [31]: 151).

θ is *logically indiscernible* from θ' in the type $\sigma =_{df}$
for each $\mathbf{f}$ $K_{log,<\sigma,\ \mathbf{t}>,\ F}$, $\mathbf{f}(\theta) = \mathbf{f}(\theta')$.

2.4. Tarski logical types

A *Tarski logical type* is a type in which all tokens are logical enti-

[14] Evidently, the numerical quantifications of the clauses 5-8 are definable only relatively to a finite universe. Moreover, the neutral quantifications of the clauses 9-12 are the quantificational versions of the four unary truth-functions.

ties. Thus, τ is a *Tarski logical type* iff $K_{log,\ \tau,\ F} = K_{\tau,\ F}$[15]

2.5. Frame changes

By definition, the set of *Tarski logical types* is invariant under any possible change of frame, and, therefore, of ontological system. The *logical notions* as such have the same property. Nevertheless, as we mentioned in the Introduction, the set of logical entities in each type (logical or not) may vary, depending on frame changes, only in a systematic way. Thus, to each logical notion N, a non-injective choice function, φ_N, may be associated, such that for each frame, F, $\varphi_N(F)$ is the logical entity generated by F which falls under N in F. For instance, φ_{True} , φ_{False} , φ_{Idrel} -σ , φ_{Univ} -<q, t> , φ_{Null} -<q, t>, φ_{Univ} -<σ, t>, φ_{Null} -<σ, t>, are the choice functions associated, respectively, to the logical notions of *True*, *False*, the *Identity relation in the type* σ, the *Universal* (*Null*) *proposition*, (the function which assigns **T** (**F**) to each sequence), the *Universal* (*Null*) *property of type*-σ *entities* (the function which assigns **T** (**F**) to each entity of this type). We will also say that a logical notion, N is a *truth-notion* iff there is a type σ T_{log}, such that for each frame, F, $\varphi_N(F)$ **:** σ.

φ_N is not-injective, because there is at least one logical notion, *False*, whose associated function, φ_{False}, has the *empty set* as trans-frame constant value. With regard to the logical notion, *True*, and its associated function, φ_{True}, there are - we have said - two main possibilities, depending on the mode of representing the entities which fall under this notion (either as the universe of the frame, or as { }), with important consequences for some other logical notions.

(1) If, in general, $\varphi_{True}(F)$ = UNIV(F) (as in Tarski), then for each logical notion, N, different from *False*, $\varphi_N(F) \neq \varphi_N(F')$, provided that UNIV($F'$) ≠ UNIV($F$), and therefore SEQ($F'$) ≠ SEQ($F$), and acc($F'$) ≠ acc($F$).

(2) If, otherwise, $\varphi_{True}(F)$ = { }[16], then for all frames F, F': (i) for

---

[15] Therefore, the set T_{log} of *Tarski logical types* coincides precisely with the set T_{tv} of types of *truth-value based entities*.

[16] It is necessary to stress that even if $\phi_{True}(F)$ = { }, there is other logical notion, *to be an Universe*, such that for each F, $\phi_{Univ.}(F)$ = UNIV(F).

each truth-notion, N, $\varphi_{TN}(F) = \varphi_{TN}(F')$, but (ii) for each logical notion, N', which is not a truth-notion, if UNIV(F) $\neq$ UNIV(F'), then $\varphi_{N'}(F) \neq \varphi_{N'}(F')$.

The ontology of total functions presented here excludes that the *same entity* is logical with respect to one frame and non-logical with respect to another. If an entity is logical in an ontological system, then, either it persists in all the other ontological systems having the same typology or, simply, it does not appear, *as such*, in these systems. For different universes always induce different functional constructs, even when they share some individuals. On the other hand it is possible, for instance, that if UNIV(F) UNIV(F') and N is a functional logical notion, then, although $\phi_N(F)$ ($\phi_N(F')$) does not appear in the ontological system generated by F' (F), $\phi_N(F')$ is a *conservative expansion* of $\varphi_N(F)$ or, alternatively, $\phi_N(F)$ is the *restriction* of $\phi_N(F')$.

To conclude: for each σ T, each logical notion, N (in σ), each frame F, and each θ $K_{\sigma, F}$, if $\phi_N(F) = \theta$, then for each frame F', there exist exactly a θ' $K_{\sigma, F'}$ such that $\varphi_N(F') = \theta'$, where θ' can be the same or a different entity than θ, and if θ' is different from θ, either θ' is set-theoretically connected in some way with θ, or both are totally independent entities.

II. THE CONTEXT OF LOGICALITY PROBLEM

1. Classical logicistic program and logical constants

The main goal of this section is to sketch the philosophical and scientific backgrounds of Tarski ontological criterion of logicality. The invariance of logical expressions of the simple type theory under permutations of the domain of interpretation was suggested by Lindenbaum and Tarski in the paper "On the Limitations of the Means of Expression of Deductive Theories", ([15]):

> "Roughly speaking, (…) every relation between objects (individual, classes, relations, etc.) which can be expressed by purely logical means is invariant with respect to every one-one mapping of the 'world' (i.e. the class of the individuals) onto itself and this invariance is logically provable.

> The theorem is certainly plausible and had already been used as a premise in certain intuitive considerations. Nevertheless it had never precisely formulated and exactly proved." ([15]: 385).

This remark, however, does not mean that Tarski already recognised the permutation invariance as the criterion of logicality. In his 1936 article "On the Notion of Logical Consequence" Tarski claims that the division of the class of all expressions into logical and extra logical is arbitrary, expressing hope for the possibility of finding the right criterion for that division:

> "no objective grounds are known to me which permit us to draw a sharp boundary between [logical and extra logical expressions] (...) Further research will doubtless greatly clarify the problem (...)" ([29]).

Tarski's criterion of logicality, which we have analysed in this paper, was formulated in the lecture he gave in 1966, nine years after the publication of Mostowski´s paper "On an Generalisation of Quantifiers" ([22]), where the criterion of permutation invariance was used to define the class of logical quantifiers. Thus this criterion should be properly called "Mostowski-Tarski criterion"[17].

The distinction between logical and extra logical constants has been in fact well known for ages as the traditional distinction between categorematic and syncategorematic expressions. In traditional syntax of syllogistics, the logical constants, i.e. the syncategorematic expressions (*all*, *none*, *some*, *some-not*) have their special positions in categorical sentences (A, E, I, O) (vid. Englebretsen [8]). Nevertheless, the problem of an objective criterion of logicality had never been raised until Frege. One of the features of his paradigm was the total change of

---

[17] There is a tradition which attributes the precise formulation of the criterion to Mostowski ([22]), but it is a little injustice to Mautner who in 1946 had considered logic as an "invariant-theory of the symmetric group...of all permutations of the domain of individual variables." ([18]: 345). In fact, Mostowski himself cited Lindenbaum-Tarski and Mautner [22], p. 13. On the other hand note 2 of the Lindenbaum-Tarski paper, probably added in the 1956 edition of *Logic, Semantics and Metamathematics*, remits, for a "detailed proof of Th. 1", to a book written by Mostowski in 1948: *Logika Matematyczna*, Warszawa-Wroclaw. Perhaps, an irenic interpretation of this tangle can be simply that the criterion of logicality was an idea which hang about Tarski's school during the thirties and forties, although it did not attain a clear recognition until 1957.

traditional syntax of the language of logic. In the new Fregean syntax of logic two groups of expressions are present: *functors* as unsaturated expressions and *arguments* as saturated expressions and there is no special place for logical constants.

There was yet another problem related to the growing interest towards logical constants in the first half of the XXth century. Nearly all philosophy of logic in this period had to deal with the so-called *logicistic program*. The logicistic program, or *logicism*, was the program of reduction of mathematics, or at least some of its branches, to logic. The program was supported by many philosophers from Leibniz to the members of the Vienna Circle. Leibniz maintained that all mathematics can be deduced from simple equalities, in finite numbers of steps, using only definitions an valid rules of inference. On the other hand, Immanuel Kant treated mathematical truths as *synthetic a priori*. Using modern terminology, we could say that according to Leibniz all mathematical constants (e.g. natural numbers) were (in some sense) logical, and, according to Kant, extra logical. Frege, who supported Kantian point of view on geometry, maintained that all arithmetical theorems could be deduced from logic. Though the program initiated by Leibniz did not manage to justify its main theses, many results obtained by logicistic-minded authors are recognised as very important to mathematics, logic and philosophy. Even if one did not agree with the logicistic program, he would have to admit that the problem of the borderline of logic and mathematics needs at least some justification. One of the ways of dealing with this task was to analyse the concepts of *logical constants* and *mathematical constants*, and to introduce sharp criteria of dividing expressions that appear in mathematical language into *logical* and *extra logical* ones. We will comment shortly three classical approaches.

Russell

Russell was probably the first to express some intuitions on *logical constants*. In *The Principles of Mathematics*, 1903, he writes:

> "The logical constants themselves are to be defined only by enumeration, for they are so fundamental that all the properties by which the class of them might be defined presuppose some terms of the class." ([26]: 8-9).

There are, however, some traces suggesting that Russell was at least ready to accept the existence of criteria of logicality. For Russell, logic was essentially concerned with "inference in general", and he did not exclude the possibility of formulating the criteria of logical constancy, e.g. in terms of rules of deduction:

> "What symbolic logic does investigate is the general rules by which inferences are made, and it requires a classification of relations or propositions only so far as these general rules introduce particulars." ([26]: § 12).

In a draft of *The Principles of Mathematics*, Russell proposes to define the concept in a way resembling Tarski:

> "And logical constants are classes or relations whose extension either includes everything, or at least has as many terms as if it included everything. And a collection has as many terms as if it included everything, when there is a relation which every possible term, without exception, has to one and only one term of the given collection, provided that to every term of the given collection some term has the given relation". (cit. in [3]: 359).

From the quotations we see that, apart from its declared agnosticism, Russell hesitated between two possible ways of explication of the concept of the logical constant in terms of the proof theory and in terms of semantics or ontology. In fact, almost all further investigations on the subject concentrated on the two mentioned approaches.

Carnap

The notion of logical constant and the criteria of logicality constituted an unsolved problem in Carnap's main works where he tried to explicate the concept of *analyticity*. The division between analytical and non-analytical sentences, in the strict, *explicated* sense of these terms, presupposes the division between logical and extra logical constants. In his *The Logical Syntax of Language* ([4]), he adopts a pure syntactical criterion. As logical constants, or rather logical expressions, he reckoned *numerical constants*, *numerical variables*, *individual variables*, *sentential connectives*, *parenthesis*, and *some operation symbols*. In his *Introduction to Semantics* ([5]), he admitted that his previous criterion was inadequate and the proper criterion should be a semantical one. In fact Carnap was sceptical on the existence of an adequate criterion for logicality of symbols. The idea of logicality was unclear and supported by the rather vague intuition that logical symbols were *meaningless* and they had no meaning independent of the context. Moreover, some signs seemed "more logical" than others. In *Meaning and Necessity*, Carnap writes:

> “Only (declarative) sentences have a (designative) meaning in the strictest sense, a meaning of the highest degree of independence. All other expressions derive what meaning they have from the way in which they contribute to the meaning of the sentences they occur. One might perhaps distinguish - in a vague way - different degrees of independence of this derivative meaning. Thus, for instance, I should attribute a very low degree to '(', somewhat more independence to ' ', still more to '+' (...). And where to make the cut between expressions with no or little independence of meaning ('syncategorematic' in traditional terminology) and those with a high degree of independence, to be taken as designators, seem more or less matter of convention.” ([6]: 7).

Presented intuition cannot be treated as an adequate criterion of logicality. Earlier, in *Introduction to Semantics*, Carnap claimed that though the division of expressions into logical and extra logical (descriptive) can be established in a particular language, there are no general criteria of logicality of constants:

> “So far we have discussed the distinction between logical and descriptive expressions only in a form in which it appears when we have to do with a particular semantical system, in other words, as a question of special semantics. The problem is more difficult in the form it takes in general semantics. Here it is the question whether and how 'logical' and 'descriptive' can be defined on the basis of the other semantical terms, e.g. 'designation' and 'true', so that the application of the general definition to any particular system will lead to a result which is in accordance with the intended distinction. The satisfactory solution is not yet known.” ([5]: 59).

Reichenbach

In his approach to natural language, Reichenbach used the standard *functor-argument* technique of formal logic, very far from the syntax of natural language. Nevertheless, his results were not trivial. For Reichenbach the problem of the criteria of logicality of expressions in natural language is even more important than in formal logic, where it is solved arbitrarily.

His analysis of natural language, (cf. [25]) starts with a criticism of traditional grammar. In Reichenbach's opinion, the right division of parts speech should be the following: argument terms, functional terms, and logical terms. All terms are either denotative or expressive. Expressive terms may express relations between functors and their arguments, e.g. the functor-argument relation “is” in the sentence “Peter is tall”, but they denote nothing as “(…)” in “T(p)”. Of course, “is”, or its symbolic counterpart, “(...)”, can be substituted by a second-order denotative term: “is in an argument-functor relation with”, as in “Peter is in an argument-functor relation with tallness”, or $\alpha(p, T)$. Also, the formula “ xP(x)” may be substituted by “Univ (P)” which means that the property P is universal. So every logical constant is usually an expressive term, or can be replaced by a denotative term in a higher-order language. In other words, *syncategorematic symbols can be substituted by categorematic ones*. This observation illustrates a serious problem: all denotative counterparts of (logical) expressive terms loose their privileged position and can be reinterpreted in Tarski's definition of *logical consequence* as all extralogical denotative terms do. One possibility to eliminate this problem is to give a semantic or ontological criterion of logicality. Reichenbach choose yet another way. He formulated the so-called “*Notational Principle*”:

> “When a formula can be verified without empirical observation, it must be possible to write the formula in such a form that, if all the denotative constants are replaced by corresponding variables, the formula remains true.” ([24]: 324).

Thus, the second-order denotative terms as “Univ” are logical because it is possible to reformulate analytical sentences containing them in such a form where their expressive counterparts do occur (vid. for an example, [25]: 324). The expressive character of terms can be analysed on a syntactical, semantical or pragmatical level. Reichenbach divides logical terms of natural language into: logical terms of a syntactical capacity, logical terms of a semantical capacity, and logical terms of a pragmatical capacity. The list of logical terms of natural language he gives is very impressive, but his criteria are not far from the traditional scholastic division onto categorematic-syncategorematic.

2. Recent approaches to logical constants

In the second half of the XXth century, the formulation of criteria of logicality of constants was even more important than at the beginning of the century. There were al least two reasons for this. First, non-classical logics appeared, whose “logical status” was somewhat suspected. The anxiety about the growing number of “logics” has been expressed e.g. by Steven Kuhn ([14]), who stated that various intensional theories in fact do not belong to logic. One of the sources of the misunderstanding was, in Kuhn's opinion, the mistaken identification of the notion of *constancy* and the *logicality* of expressions. Quine, in his *Philosophy of Logic* ([24]), restricted the proper logic to the first-order level. According to him, the second-order logic presupposes some extra logical distinctions, e.g. quantification over individual variables *vs.* quantification over predicative variables. On the other hand he maintains that at least some of the first-order non-classical logics result from different explications of natural language counterparts of logical expressions.

To solve the problem of logical constants various criteria have been used. For instance, Peacocke ([23]) formulated his criterion of logicality of expressions in terms of the *a priori* knowledge. Another approach is due to Hacking ([11]) and Dosen (mainly in [7]). In their opinion, logic is the theory of logical inference, and the criteria of the logicality of constants should be expressed in its terms. The most general theory, or rather metatheory, of logical inference is Gentzen's calculus of sequents, with clear distinction between structural rules and the rules of introduction of logical constants. Structural rules alone express the properties of logical inference. The criteria of logicality of constants are based on the supposition that the rules that introduce them add nothing new to logical inference. Hacking stated that a given constant introduced to the Gentzen system of a given language is logical, if after introducing it the structural rules of the enriched language can be eliminated. First of all, the *cut rule* should be totally eliminable, as in the "pre-logical language" of the structural rules only (i.e. without logical constants) the *cut rule* is redundant. The *axiomatic rule* and the *dilution rule* should be reducible to their versions without logical constants. Dosen´s approach consists in adapting Moore's standard notion of *analysis*, and stating that an introduced constant is logical if it is analysable in the language of structural rules alone. He also formulated the thesis that classical, intuitionistic and relevant logics have the same logical constants although introduced in different structural contexts, i.e. with different restrictions on structural rules.

Other criteria of logicality are semantical or ontological. Apart from formulation differences, they state that denotations of logical constants can be the entities invariant under the permutations of the domain of interpretation and express the intuition called sometimes context-independence or topic-netrality. The publication of [22] is commonly considered as the beginning of *abstract model theory*. It has been cited by almost all authors investigating the semantics of natural language determiners. The application of the permutational invariance criterion was possible in [22] because the quantifiers were treated categorematically, i.e. as the members of an appropriate syntactical category for those of the first-order language. The criterion permitted not only the introduction of the quantifiers *definable* in first-order language with identity, but also the introduction of some *indefinable* quantifiers, e.g. "for finitely many". The criterion of permutational invariance was then adopted by Per Lindstrom ([16]) who extended the notion of quantifier by introducing quantifiers of various types into the first-order language. Some years later the same criterion was used by Timothy McCarthy ([19]) to show that all logical expressions of the standard first-order language are permutation invariants. Tarski's final version of the criterion with the application to the entities of various categories was formulated in 1966, but it remained practically unknown until its publication by John Corcoran twenty years later.

The Mostowski-Tarski criterion turned out to be extremely useful in finding the "logical core" of natural languages, and especially in those parts that cannot be expressed in the first-order language of logic. The model-theoretic approach to the semantics of natural language has been systematically initiated by Montague in the early seventies. Determiners such as e.g. "all", "some", "a", "the", "for at least seven", or "for exactly three" were strictly translated by corresponding quantifiers of the first-order language with the aid of the lambda operator and the typed intensional logic. But, though Montague's results in model-theoretic semantics of natural language are recognised as very important and innovative, he did not formulate the criteria of logicality. The permutation invariance was used in 1981 in Barwise and Cooper's article "Generalized Quantifiers in Natural Language" ([2]). This paper has initiated very intensive investigations on natural language determiners, their various classifications and types of monotonicity. This classification helped to find "logical gaps" in natural language, i.e. to find entities which are possible denotations of non-existing determiners. The most significant papers on the topic are written by van Benthem ([33] and [35]), Westertahl ([37]) and Keenan and Stavi ([13]). The Mostowski-Tarski criterion has been gradually recognised as the necessary condition not only for the logicality of quantifiers or determiners, but for expressions of other categories as well. In particular, Westertahl [38] and van Benthem [34] sought the logical items of various types as the possible denotations of logical expressions of different categories of natural language. Their results concerning logical entities of different types are very similar to those obtained in our paper. The difference lies in the kind of semantics (formal ontology) that has been applied. Our use of the notion of sequence of objects of the universe, is much closer to McCarthy´s approach than to that of Westertahl or of van Benthem.

University of Santiago of Compostela, lflpvill@usc.es
University of Łódź, janmac@krysia.uni.lodz.pl

REFERENCES

[1] Aczel, P. “Replacement Systems and the Axiomatization of Situation Theory”. En Cooper, Mukai-Perry (eds.), *Situation theory and its Applications*, C.S.L.I. Stanford, 1990: 3-31.

[2] Barwise, J., Cooper, R., “Generalized Quantifiers and Natural Language Determiners”, *Linguistics and Philosophy*, 4, 1981, 159-219.

[3] Byrd, M., “Russell, Logicism, and the Choice of Logical Constants”, *Notre Dame Journal of Formal Logic*, 30, 1981, 343-361.

[4] Carnap, R., *Logical Syntax of Language*, Harcourt and Kegan, New York, 1937.

[5] Carnap, R., *Introduction to Semantics*, Harvard U.P., Cambridge, MA, 1942.

[6] Carnap, R., *Meaning and Necessity*, University of Chicago Press, Chicago, 1957.

[7] Dosen, C., “Logical Constants as Punctuation Marks”, *Notre Dame Journal of Formal Logic*, 30, 1981, 362-381.

[8] Englebretsen, C., “Formatives”, *Notre Dame Journal of Formal Logic*, 30, 1981, 382-389.

[9] Etchemendy, J. , “Models, Semantics and Logical Truth”, *Linguistics and Philosophy*, vol. 11, 1988), 91-96.

[10] Etchemendy, J. , *The Concept of Logical Consequence*, Harvard University Press, Harvard, 1990.

[11] Hacking, I., “What is Logic?”, *The Journal of Philosophy*, 76, 1979, 285-319.

[12] Keenan E. L, Faltz, L., *Boolean Semantics for Natural Languages*, Reidel, Dordrecht, 1985.

[13] Keenan, E.L., Stavi, J., “A semantic Characterization of Natural Language Determiners”, *Linguistics and Philosophy*, 9, 1986, 253-326.

[14] Kuhn, S., “Logical expressions, Constants and Operator Logic”, *The Journal of Philosophy*, 78, 1981, 487-499.

[15] Lindenbaum, A. and Tarski, A., "On the limitations of the means of expression of deductive theories", 1934-35), in Tarski [10]: 384-392.

[16] Lindstrom, P., "First Order Predicate Logic with Generalized Quantifiers", *Theoria*, 32, 1966, 186-195.

[17] Maciaszek, J. and Villegas-Forero, L. "A note on compositionality in the first order language", *Bulletin of the Section of Logic* (University of Łódź. Poland), Vol. 24, n. 4, 1995, 206-214.

[18] Mautner, F.I., "An Extension of Klein's Erlangen Program: Logic as Invariant-Theory", *American J. Math*. 68, 1946, 345-384.

[19] McCarthy, T. , "The Idea of a Logical Constant", *The Journal Of Philosophy*, vol. 78, pp. 499-523.

[20] Montague, R. , "Pragmatics", in [30]: 95-118.

[21] Montague, R. , "Universal Grammar", in [30]: 223-246.

[22] Mostowski, A., "On a Generalization of Quantifiers", *Fundamenta Mathematicae*, 44, 1957, 12-36.

[23] Peacocke, Ch., "What is a Logical Constant?", *The Journal of Philosophy*, 73, 1976, 221-240.

[24] Quine, W.V.O., *Philosophy of Logic*, Prentice-Hall, New Jersey, 1970.

[25] Reichenbach, H., *Introduction to Symbolic Logic*, McMillan, New York, 1947.

[26] Russell, B., *The Principles of Mathematics*, Allen & Unwin, London, 2nd ed. 1950.

[27] Sher, G., *The Bounds of Logic*. The MIT Press, Cambridge, MA, 1991.

[28] Simons, P., "Bolzano, Tarski and the limits of the Logic", In P. Simons, *Philosophy and Logic in Central Europe from Bolzano to Tarski.*, Kluwer, Dordrecht, 1992, chap. 2.

[29] Tarski, A., "On the Notion of Logical Consequence" [1936], in Tarski [27]: 409-420.

[30] Tarski, A. , *Logic, Semantics and Metamathematics*, 2nd ed. (ed. Corcoran, J). Hackett, Indianapolis, 1983.

[31] Tarski, A., "What are Logical Notions?" (edited by J. Corcoran), *History and Philosophy of Logic*, 7, 1986), 143-154).

[32] Thomason, R. (ed.), *Formal Philosophy. Selected Papers of R. Montague*, Yale University Press, New Haven, 1976.

[33] Van Benthem, J., "Questions about Quantifiers", *The Journal of Symbolic Logic*, 49, 1984, 443-466.

[34] Van Benthem, J., *Essays in Logical Semantics*, Kluwer, Dordrecht, 1986.

[35] Van Benthem, J., “Logical Constants Across Various Types”, *Notre Dame Journal of Formal Logic*, 30?, 1989, 316-342.

[36] Villegas-Forero, L. and Maciaszek, J., “Formal Ontology, Semantic Interpretation and Logic”, in C. Martínez Vidal, U. Rivas and L. Villegas-Forero (eds), *Truth in Perspective. Recent issues in Logic, Representation and Ontology*, to appear in Ashgate (Avebury Pub.), England.

[37] Westertahl, D., “Some Results on Quantifiers”, *Notre Dame Journal of Symbolic Logic*, 25, 1984, 152-170.

[38] Westertahl, D., “Logical Constants in Quantifier Languages”, *Linguistics and Philosophy*, 8, 1985, 387-413.