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NATURAL LANGUAGE CONDITIONALS[1]

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Abstract

This paper introduces a formal theory of natural language conditional statements. Traditionally, the best models for conditional statements have been defined in the field of possible world semantics (Stalnaker, Lewis, Pollock, Nute), but all of them failed to give a complete account of the set of relevant conditional statements belonging to a standard example of natural language (as, for example, English). The possible world approach owes this fault not to lack of formal expressiveness, but to a wrong pretheoretical setting apart of the different kinds of conditionals. The theory presented here bases the classification not in the verb of the antecedent (indicative or subjunctive) but in the "implicational strength" expressed by the conditional: the degree to which compliance with the antecedent of a conditional sentence ensures satisfaction of the consequent. Four conditional operators are defined, and some novel features are introduced in the semantics, in order to give an interpretation to these operators.

Key Words: Modal logic, Conditionals, Similarity Relation, Possible Worlds Semantics.

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1. Introduction[2]

Natural language conditionals are one of those "selected topics" that every general introduction to logic must include. Not only because they constitute one of the most difficult puzzles that any logic paradigm may face up to, but also because they are a perfect example of the high degree of sophistication that our formal tools must reach in order to solve problems involved by natural languages. In spite of the virtues of both the metalinguistic (Chisholm, Goodman, Mackie) and the probabilistic approach (Adams), I believe that the theories built from possible world semantics are the best fitted to our linguistic intuitions about the properties and truth-conditions of conditional statements. When testing a conditional, we resort to situations where the antecedent of the conditional holds, and we seek for the truth-value of the consequent on these situations. This idea is easy to grasp by using a set of possible worlds as the situations where the conditional is tested. I will call the set of possible worlds where a concrete conditional is tested its "evaluation universe". Furthermore, possible world semantics explains quite well the modal feature of conditionals, which we may call "conditional necessity". "Conditional necessity" means that, when the antecedent-true condition is fulfilled, the consequent is in a (conditional) sense necessary: it holds in the whole set of relevant possible worlds that constitute the evaluation universe. One of the most important points in the theory of conditionals is the definition and characterisation of the evaluation universe for standard conditional statements. Since the pioneering work by Stalnaker and Lewis, it is usual to use the notion of similarity to define the evaluation universe: we take as the evaluation universe for a given conditional the set of possible worlds similar in some degree to the actual one. The problem is now transferred to the explanation of our standards for considering a possible world "similar enough" to the actual one. In this way, we close the hermeneutic circle for conditionals: the conditional necessity of the consequent is explained in terms of the evaluation universe, and the universe evaluation is explained in terms of the similarity relation between possible worlds. The higher the degree of

[2] This paper focused on the conceptual and philosophical aspects of natural language conditionals. Consequently, I will not pay much attention to the formal aspects of the theory given here. The formal theory is fully provided and explained in Vilanova [23], where the logical system presented here is defined as one designed element of a class of logics that I call "multiconditional logics". I refer also to Vilanova [23] for metalogical results as completeness and soundness, which are not proved here. Some proto-versions of this theory were given in Vilanova [19] and Vilanova [20], although till this very moment I had not made a complete exposition.

conditional necessity, the wider the evaluation universe, and the weaker the standards of similarity.

Promising as this poising of the problem is, the possible worlds approach fails to give an exhaustive theory of natural language conditionals. The discussion has been focused on the question about the standards of similarity (maximal, minimal, high or low) used in the testing of conditionals, but none of the propounded solutions seems to comprise the whole set of relevant conditional statements. In my opinion, the reason of this incapacity is that we use more than one standard of similarity when testing conditionals. In fact, my thesis is that there are different kinds of conditional statements corresponding to different standards of similarity. Consequently, an exhaustive theory of natural language conditionals must proceed by characterising the relevant different standards of similarity, which correspond to the different conditional operators we need in our theory. I will provide arguments for this theory afterwards. But, first of all, I want to take a look at the traditional theory of conditionals.

2. Two kinds of conditionals

Traditionally, the key for classifying conditionals has been the mood of the main verb in the antecedent of the conditional. On the one hand, we have the conditionals where the main verb of the antecedent is in subjunctive mood, for example:

(2.1) If the window were open, the temperature would be lower.

The former statement says that *in the present situation*, if someone happened to open the window, then the room would be colder. The standard examples of subjunctive type conditionals express an accidental, momentary fact, as contingent as for example the day being cold and rainy or the heating system good operation. Furthermore, their truth value depends strongly on the context, so the implicational relation between antecedent and consequent is closely related to the actual state of things.

On the other hand, we have the conditionals where the main verb of the antecedent is in the indicative mood. For example:

(2.2) If it rains, the streets are wet.

Some weak form of the subjunctive-type conditional may be expressed in the indicative mood, when the relation between the antecedent and the consequent is that logically symbolised by material implication. But

I doubt that this is more than a very exceptional and limited case[3]. In general, indicative conditionals applies to more than one case. The former one states that, whenever it rains, the streets are moistened. They do not depend on the context as much as subjunctive conditionals, and, furthermore, they are more solid, stable and, in a certain sense, universal: whatever they express is pretended to hold in any occasion unless, perhaps, some extraordinary combination of events occurs[4].

I consider Burk's calculus for causal statements a good example of a model for this kind of conditional. For Burks, the former statement would be translated into "$A \rightarrow^{c} B$", which means, by definition $\Box^{p}(A \rightarrow B)$. "$\Box^{p}A$" means that A is physically necessary, that is to say, A is true at every world which shares our world physics (every physically possible world). A certain sophistication in our semantics is now required: we need not only the set of (logically) possible worlds but also one of its subsets, the set of physically possible worlds. Actually, we must have not only a such subset for the actual world, but for every possible world, so we arrive to a S5 modal logic with a semantic based on a reflexive, transitive and symmetric (physical) accessibility relation between possible worlds.

A typical model for subjunctive conditionals is Lewis' VC logic for counterfactuals. The subjunctive mood in English have the sense of hypothetical, not real. That means that the antecedent of the subjunctive conditionals expresses something that in fact does not happen in the real world, something that is not factual, but counterfactual. For this reason, authors like Lewis call these statements "counterfactuals". Lewis' operator truth conditions are a formal translation of Ramsey's test for counterfactuals, which express a procedure that usual speakers of a language use to know the truth value of a conditional whose antecedent is false:

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[3] In spite of David Lewis' point of view, I cannot think that when we use the words "if A then B" in indicative conditionals, we want to say just that It is not the case that A and not B. This point will be proved afterwards.

[4] Perhaps someone will feel that this is a very coarse simplification of the problem. Actually, investigations in this field have provided a variety of formal models for conditionals that *stricto sensu* ought to correspond to different types of natural languages conditional structures. But I believe that all of them are trying to modelize, in a more or less explicit way, one of these two general types (and the matter of controversy, then, is what model is the correct one for any of the two kinds of conditional. That is the point of view of, for example, William Harper [8], pp. 19-20, Donald Nute [14], p. 389, and Lewis [11], p 5. Anyway, if anybody thinks that this classification is not satisfactory I will tell him that I agree completely, because, as you will see later, my aim is to show its inconvenience.

(D 1.2) A → □B is true at a world *i* if and only if
(a) no A-world (a world where A holds) belongs to S_i (the set of worlds accessible from *i*), or
(b) there is a A-world *k* in S_i such that for any world *j*, if $j \leq_i k$ (*j* is as least as similar to *i* as *k*) then B holds at *j*.

A further sophistication in our semantics is now required: a family of (comparative) similarity relations, one for any possible world *i*, which distributes the accessible worlds around *i* according to its likeness or similarity with *i* [5].

The subjunctive conditional is true at *i* if and only if there is an antecedent world at least as close to *i* as a certain accessible world. In plain words: "A□→B" is true if in the most similar worlds to the actual one where A is true B is true.

	Context-dependent	Covering more than one situation	Related to the actual state of things	Closely related to the world physics	Stability
INDICATIVE	-	+	-	+	+
SUBJUNCTIVE	+	-	+	-	-

3. Implicational strength

If the former account is right, then English conditionals may be reduced to two groups: the indicative type will be formalized with a Burk's type operator, and the subjunctive type will be formalized with a Lewis' type one. Of course, some strange or abnormal cases will be left out (meaningless, metaphorical, orders...), and some special subclasses will need derived operators. For example, the "even if A, B" type, whose logical form looks to be "B and A□→B" is one of the subtypes. "Counteridenticals", "even-if" or "might" conditionals are other secondary cases, but it is believed in general that they can be analyzed in terms of the subjunctive or indicative conditionals.

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[5] Of course Burks' model is not the only one for the indicative conditionals, as well as Lewis' model is just one (perhaps the most celebrated altogether with Stalnaker's S2) out of a lot of similar approaches. I select them just as two good examples which can illuminate my explanation. Equally it is for the sake of clarity that I select comparative similarity relation systems instead of spheres of similarity or selection function systems.

The previous picture may look like clear and, for many, a good and reliable basis to start any investigation about conditionals. It would be this way if some "subversive" sentences did not arise. Consider, for example, the next one:

(3.1) If Peter eats chocolate, he falls sick.

And, please, accept the supposition of Peter being allergic to chocolate. Which conditional operator must we use for its formal translation? Obviously it is not Burk's operator: Peter being allergic to chocolate is not physically necessary, so it becomes false. Is (3.1) a forgery of indicative conditional and, then, a subjunctive conditional in disguise? If we use Lewis' operator it becomes true, but one cannot avoid the feeling that "$A \Box\!\!\rightarrow B$" says less than "If Peter eats chocolate, Peter falls sick". This statement, like (2.2), applies to more than one situation: what it means is that not only now but in any moment when Peter eats chocolate, he falls sick. Furthermore, the relation between the antecedent and the consequent is stronger than that involved in (2.1): B is not only true in the most similar A-worlds, but in a plurality of worlds similar in some aspects to the actual. Which are these worlds is actually the battlefield for its modelling. We will come back to this question later.

Another problematic case is the next one:

(3.2) If a mobile reached the speed of light its internal clock would stop.

As you know, the antecedent, according to the theory of relativity, is physically impossible. Nevertheless relativistic physics itself states that if some mobile reaches this limit its internal clock stops. It looks like as (3.2) is a physically necessary fact (perhaps a special kind of physical law) as well as (2.2), no more but no less. I will call this kind of conditional statements "substantive counterfactuals", by opposition to the accidental or contingent counterfactuals as the sentence (2.1)[6].

How can we model it? Once again, Burk's operator looks too coarse: the statement is vacuously true because in no physically possible world the antecedent can be true[7]. Just like "if the mobile reaches the

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[6] Substantive counterfactuals are a special class of counterlegals. Actually "substantive counterfactual" express a stronger concept than "counterlegal": not only the antecedent is physically impossible (as in counterlegals) but also the conditional statement *in toto* means a physically necessary fact.

[7] Burks itself is aware of this fact and, in order to avoid this problem, introduce "the non superfluous causal implication" (" $\underline{nsc\rightarrow}$ "). "A $\underline{nsc\rightarrow}$ B" if and

speed of light then everybody in the universe is a rabbit". It is difficult to know if (3.2) would be true or false formalized with Lewis operator: it depends on the similarity relation, which in Lewis' theory remains undefined (only formal requirements are given). Anyway, once again the relation between antecedent and consequent looks stronger than what Lewis' operator can express.

These examples suggest that the indicative/subjunctive distinction is too coarse to grasp the important different cases of conditionals. Moreover, not only there are problems to know if one statement fulfils the properties of a certain type or must be modelled with a given operator, but also it is difficult to distinguish if the statement (well) formalized with a operator would be true. As, for example, with (3.2.) and the Lewis' operator.

These problems point out that there are more significant cases of conditional necessity than the two extremes that examples (2.1) and (2.2) represent. They show, also, that the logical difference between conditional statements lies in the degree of conditional necessity, and not in the hypotheticality or reality of the antecedent. What we need to define is several "implicational strengths" that can be expressed by the idiom "if, then". By "implicational strength" I mean the degree to which compliance with the antecedent of a conditional ensures satisfaction of the consequent. In other words, the implicational strength of a conditional statement is the degree of conditional necessity of its consequent when its antecedent is true. I feel that four significant implicational strengths (apart from logical consequence) may be found in English language. I will define them afterwards. At this very moment, I just want to give an example. Suppose that John, who is allergic to chocolate, is at this moment standing at the edge of a precipice. The following sentences exemplify the four different implicational strengths:

(3.3) If Peter steps forward, he will die.
(3.4) If Peter eats chocolate, he will die.
(3.5) If Peter eats cyanide, he will die.
(3.6) If Peter stops living, he will die.

As we said before, the degree of conditional necessity depends on the size of the evaluation universe, which depends in turn on the standard of similarity. So, if we want to characterise the types of conditional strength, we need to provide different degrees of similarity required for

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only if A implies causally B, A is not physically impossible, B is not physically necessary and A and B are logically independent (the three possible motives of triviality). But the non superfluous causal implication is not a satisfactory solution: all substantive counterfactuals are now (trivially) false.

the worlds to belong to the evaluation universe. How can we do it? It is obvious that, if we want to make those distinctions *into* the similarity standards themselves, we need a definition of the procedure to calculate if a world is as similar (or more, or less) to the actual one as a third one. But this is impossible to get if we use the vague, *a priori* definition of Stalnaker, Lewis and others. In effect, they introduce the similarity relation as a primitive, and only provide some formal constraints, but we need something more. Authors as Ginsberg and Smith, or Thomas Mormann have tried a theoretical exploration of the concept with success, giving criteria and procedures to calculate concrete similarity relations. In the next paragraph, I will suggest a generative definition of the similarity relation that, I believe, corresponds to the way English speakers compare alternative situations during the testing of conditionals.

4. The similarity relation

The basic notion in possible world semantics is that of a model as a pair <**K**,[ ],> being K the set of possible worlds and [ ] an interpretation which assigns to any sentence of the language a subset of K (the possible worlds where the sentence holds). Since K is seen as the whole set of different alternative situations to the actual one two requirements must be fulfilled:

> (DEF. 4.1.) A *possible worlds* model is a pair <K,[ ]> where K is a set of possible worlds {*i*,*j*,*k*…} and [ ] a function from K to **P**(K) such that:
>
> (K.1) [ ] is an interpretation of sentences over K which makes all tautologies valid;
>
> (K.2) For every *k*, *j* belonging to K, there is at least one atomic sentence A of the language such that *k*   [A] and *j*   [A], or *k*   [A] and *j*   [A];
>
> (K.3) For every pair of atomic sentences A, B of the language there is at least one *i*   K such that *i*   [A] and *i*   [B], or *i*   [A] and *i*   [B];

In this paragraph a sophistication of this notion of model will be developed in order to give truth conditions for conditional operators. The point of departure is the intuition of the actual world[8] not as a sim-

[8] You may understand by the “actual world” any system or situation we want to modelize: the whole world is just a border (and possibly more ideal than real) case of the applications of the model.

ple "collection of facts" but as a structured entity, that contains some elements more important than others, and then have properties more "essential" or more "accidental" than others. The linguistic description of the world must reveal this structure, assigning to every statement its ontological or epistemological status. From an intuitive point of view, when evaluating a conditional statement we count not on the world itself (and then, not on an exhaustive and recursive truth function of all the possible statements about the world[9]) but on a basic and stratified description of the world. I think that four kinds of statements must be distinguished in this stratified description. Every strata plays a particular role when evaluating conditionals or calculating the degree of similarity between worlds: conceptual, physical, actual and purely contingent statements. I will discuss each of them separately.

I mean by semantically or analytically distinguished statements (semantical sentences in short) those sentences which express the semantical structure of the language which is used to express the world's description. In other words, those statements of the stratified description which are (necessarily) true just on the basis of the analytical relations between concepts. For example, "A widow is a woman", "Tomorrow today will be yesterday" and "Neutrons are atomic particles" are conceptual sentences. Perhaps someone will think that conceptual statements does not express a fact about the world itself but only a (contingent) convention of ours, and, in this sense, must be removed from the facts we take as basis for evaluating conditionals. But we must not forget that, once those conventions are established, one cannot consider the evaluation of any conditional statement (for example, "if the train leaves Madrid on Saturday it will arrive to Warsaw in Sunday") if the concepts involved and its analytical relations between another concepts are not fixed ("Sunday is the day after Saturday").

I mean by physically designated statements (or physical sentences) those which compound the "physics" of the world. "Physics" here means a concept wider than the physical science or the physic laws. It must be understood as the general rules which express the "working" of any possible (physical or not) system. Or, in other words, the statements that establish "the way things use to be", what facts may happen and what facts have to happen, and what are the concatenation of events which occur. It is in this sense that we can talk about the physics of a chess game, the democratic government, or a city's traffic system.

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[9] Taking account simply of this exhaustive function will lead to the (absurd) phenomenon of considering not only a sentence "A" but also "A & A", "A & A & A", "A & A & A & A", ...

Perhaps the concept of actually designated statements (actual sentences in short) is the less familiar in the study of conditionals, but I think that it is very known and used (in a more or less implicit or unconscious way) in everyday life. I mean by actually designated sentences those which express facts that express the structural properties of the entities in the world. The modal aspect of actual statements is that, unlike physical statements, they could have been false in the actual world, but, once they are true, they cannot stop being it. Think of my being diabetic in the previous example: I could have born not diabetic, but since I have born this way I will be diabetic all my life[10]. We can define actual sentences in a temporal logic, using a notion of "durability" like "true with a probability n during at least an interval of time m". n and m are numerical coefficients that may vary from one evaluation context to another. This variability explains one the most generally accepted features of natural language conditionals: their vagueness[11].

Finally, the set of purely contingent statements (or contingent sentences) are the set of sentences which complete the stratified description not being conceptual, physical or accidentally distinguished statements, and then accidental or contingent *stricto sensu*.

Formally these notions will be introduced in our system by adding to our models a third element, the function $\boldsymbol{\mu}$ that I will call "structuring function" which assigns to any world *i* a stratified description of that world *i*, $\Delta_i = \langle N_i, F_i, A_i, C_i \rangle$, being $\langle N_i, F_i, A_i, C_i \rangle$ (I will call them

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[10] I take inspiration for the concept of actually designated statement from Pollock's "actual necessity", but I use it in a rather different sense. Pollock [16], pp. 51-63, defines strong subjunctive generalizations (those which are true for all physically possible objects; "any raven would be black") and weak subjunctive generalizations (those which are true for all actually possible objects; (a) "any human which drinks from this bottle would die"; "this" bottle containing poison). Then defines an actually possible proposition as a proposition which is compatible with the set of weak and strong subjunctive generalizations, and an actually necessary proposition as a proposition whose negation is not actually possible. Besides the pragmatic aspect of my own concept, I do not agree with Pollock that actual possibility (as well as actual necessity) proceeds from generalisation about objects, and then from universal statements. From my point of view, in any case, what is actually designated in the previous example is the statement (b) "this bottle contains poison" which is what makes Pollock's statement true (and true at every actually possible situation). Anyway, I doubt whether (a) as (b) are actually designated statements. Statement (b) belongs to the set of purely contingent statements, whereas (a) is a conditional that must be formalized with the operator $\rightarrow^c$ I will introduce later.

[11] See Vilanova [21] for a explanation of this point.

"designed sets" from here on) the sets of conceptual, physical, actual and contingent statements respectively.

As you can suppose, the construction of the stratified description is a vital point in my model. The success and material correction of the model depends on the accuracy of the stratified descriptions. In this respect it must be understood that being atomic is not a requirement for a sentence to belong to any of the sets of the stratified description. From my point of view, "an electric particle is negative or positive" or "if a piece of iron is in a magnetic field, it experiences a force"[12].

In spite of its importance, the construction of the stratified description is an extra-logical matter. One cannot ask the logician for the description, like one can not ask the logician for the preference relation between sentences in epistemic logic. Logic only can give some formal constraints. These constraints limit the set of possible stratified descriptions appropriate to a situation, but they are not sufficient to determine it.

The most elemental requisite is the soundness of every set belonging to the stratified description. Furthermore, the whole stratified description must be sound too.

> (C.1) Δ_k is *coherent* iff for any sentence A it is not the case that $N_i \quad F_i \quad A_i \quad C_i \vdash A$ and $N_i \quad F_i \quad A_i \quad C_i \vdash \neg A$.

The stratified description of a world *i* is an "account" of the world *i*. Consequently, all sentence belonging to any of the designed sets has to be true in *i*. Furthermore, since it is not a requisite for the designed sets to be deductively closed, every logical consequence of any designed set must be hold in *i* too.

> (C.2) Δ_i is *materially correct* iff for any sentence A such that $N_i \quad F_i \quad A_i \quad C_i \vdash A$, $i \quad [A]$.

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[12]I think a brief explanation is advisable at this point. Obviously (conditional) statements of this kind are what I am trying to model with the help of the operators I introduce. Does it mean a circularity in my explanation? In order to avoid the suspicion I must emphasize that when we are designing the stratified description we do not try to "translate" the previous statement in the set P_i but only a very weak version of its informational content, that is to say, the one using material implication: "a piece of iron is in a magnetic field" $\rightarrow$ "it experiences a force". In the stratified description only basic sentences and sentences composed by propositional letters and classical logical constants are allowed.

Condition (C.2) ensures that Δ_i is indeed a description of *k* in the ordinary sense, all its formulae being true assertions about *i*. But it does not avoid that Δ_i is a description of a different world *k*. The next condition ensures that Δ_i is a sufficient description of *i* in that there is no other world to which it applies in full:

> (C.3) Δ_i is *unique* iff for any sentence A, (A $\quad N_i \;\; F_i \;\; A_i \;\; C_i$) $\rightarrow$ (*i* $\quad$ [A]) and there is no *l* $\quad$ *K*, $l \neq i$, such that for any sentence A (A $\quad N_i \;\; F_i \;\; A_i \;\; C_i$) $\rightarrow$ (*l* $\quad$ [A]).

The former constraints define very general properties of the stratified description. The formal model is complete at this point. Anyway, some additional constraints, with more material content, can be given. To start with, (C.3) says that Δ_i applies only to the world *i*, but it does not explain why. The next constraint relates this fact to the "completeness" or integrity of the stratified description. Given that the sentences included in the designed sets are going to be used as criteria for similarity measurements, it is important to be sure that every atomic sentence of the language or its negation is included in the stratified description, at least as a purely contingent sentence[13]:

> (C.4) Δ_k is *exhaustive* iff for any atomic sentence A, $N_i \;\; F_i \;\; A_i \;\; C_i \vdash A$ or $N_i \;\; F_i \;\; A_i \;\; C_i \vdash \neg A$.

The final requirement is about the "conciseness" of the stratified description. Since the similarity of a world *i* to another world *j* (as it will be seen later) will depend on the sentences in the stratified description of *j* which hold or do not hold in *i*, it is of the most importance that no redundant or superfluous information is contained in the stratified description. One way to settle this point is to avoid the presence of sentences that are logically implied by other sentences in the same strata. Being $CL(X_i)$ the deductive closure of X_i:

> (C.5.1) Δ_i is *strictly concise* iff for any designed set X_i and any sentence A $\quad X_i$, $CL(Xi) \neq CL(X_i \quad A)$

[13] This requirement will be fulfilled as long as we will not accept the existence of "trivial facts" in the world or the system we are modelling, facts which are irrelevant when considering counterfactual situations and its proximity or likeness regarding to the actual. This point is fundamental: if this requirement is fulfilled then no possible world k is as similar to some world *i* as i itself, and this is the reason why the axiom CS holds for the actual implication.

Nevertheless, this requirement is perhaps too strong. From the informational point of view, it is an idealisation pretending redundancy to be equal to zero. Moreover, in some cases it could interfere in certain cases with the former requirement, which oblige us to specify the truth or falsehood of any atomic sentence. If you prefer “flexibility” at this respect, it is better to use the notion of redundancy from the mathematical theory of information:

(C.5.2) Δ_i is concise iff its redundancy is as close to zero as possible.

It is easy to see that material correction implies coherency, that uniqueness implies coherency, and that exhaustivity implies uniqueness. (C.5) is a very vague constraint, so in the formal definition it is enough to include exhaustivity:

> (DEF. 4.2): Given a set *K* of possible worlds, a *stratified description* of a world *i* *K* is a sequence $\Delta_i = <S_i,P_i,A_i,D_i>$ such that S_i, P_i, A_i, D_i are sets of sentences and Δ_i is exhaustive. N= {S,P,A,D} is called the set of *strata indexes*. N is ordered by the *importance relation* < defined: S<P<A<C.

Now we are ready to provide the definition of the similarity relation for any world *i*, based on the stratified description for *i* given by the structuring function μ. The definition have the form of an algorithm to determine if a possible world *j* is at least as similar to the actual one *i* as another world *k*. It has three steps. In the first step we define the “similarity factors” corresponding to the designed sets. In the second step it is defined the relation $\succ$i: “…is more similar to *i* than…”. Finally, and based on it, we define $\prec_i$ “…is at least as similar to *i* as…”. The first idea is that any stratum of Δ_i provides a criteria for comparing possible worlds. The second idea is that each criteria has a different degree of importance when doing the comparison. According to this idea, the definition of the similarity relation proceeds by comparing worlds with respect to the coefficients of similarity, following their order of importance, and stopping when it reaches a difference. In other words, we use the set N_i to compare *j* and *k*, if there are different from this point of view we establish the relative similarity on this basis; if they are not, we use P_i to compare them, and so on with A_i and C_i; if they are equally similar to *i* regarding all the criteria then they are similar in the same degree to *i*.

> (DEF. 4.3): Given a model <*K*,[ ]>of possible worlds, three worlds *i,l,m* *K* and a stratified description Δ_i of *i*, then the

similarity relations $\succ_i$ ("to be more similar to *i* than") and $\prec_i$ ("to be at least as similar to *i* as") *induced by* Δ_i are defined as follows:

(1) For any world l K and x   {N,P,A,C}, the *x-th coefficient of similarity of l with respect to i*, $c_i^x(l)$, *is defined by* $c_i^x(l)$ = ½{A X_i / l $[X_i]$}½(In other words, $c_i^x(l)$ is the cardinality of the set of formulae of X_k^i that are false in l).

(2) def. of $\succ_i$

2.1) if $c_i^n(l) > c_i^n(m)$ then $m \succ_i l$ and not $l \succ_i m$,
if $c_i^n(l) < c_i^n(m)$ then $l \succ_i m$ and not $m \succ_i l$,
and if $c_i^n(l) = c_i^n(m)$, then:

2.2) if $c_i^p(l) > c_i^p(m)$ then $m \succ_i l$ and not $l \succ_i m$,
if $c_i^p(l) < c_i^p(m)$ then $l \succ_i m$ and not $m \succ_i l$,
and if $c_i^p(l) = c_i^p(m)$, then:

2.3) if $c_i^a(l) > c_i^a(m)$ then $m \succ_i l$ and not $l \succ_i m$,
if $c_i^a(l) < c_i^a(m)$ then $l \succ_i m$ and not $m \succ_i l$,
and if $c_i^a(l) = c_i^a(m)$, then:

2.4) if $c_i^c(l) > c_i^c(m)$ then $m \succ_i l$ and not $l \succ_i m$,
if $c_i^c(l) < c_i^c(m)$ then $l \succ_i m$ and not $m \succ_i l$,
and if $c_i^c(l) = c_i^c(m)$, then neither $l \succ_i m$ nor $m \succ_i l$;

(3) (definition of $\prec$) l,m K, $<l,m>$ $\prec_i$ iff $<m,l>$ $\succ_i$.

Note that, given a stratified description Δ_i of a world *i* belonging to a universe *K* of possible worlds, the induced relation $\succ_i$ is transitive and asymmetric, and the induced relation $\prec_i$ is transitive and strongly connected. By the requisite of exhaustivity, and the point K.3 in definition 4.1, the similarity relation is centred in the sense of Lewis, that is to say, for any *i*   K there is not another possible world *j*   K such that $j \prec_i k$.

5. Operators

We have four conditional operators: $\rightarrow^n$ (analytical or semantical implication), $\rightarrow^p$ (physical implication), $\rightarrow^a$ (actual implication) and $\rightarrow^c$ (contingent implication), each of them expressing a higher implicational strength than the next. All of them have similar truth conditions, differing only in the respective roles that the designed sets play in the evaluation procedure. Consequently, we will provide a general semantic definition, by using the variable x ranging in the set N={n,p,a,c}. Truth conditions are similar to Lewis operator truth conditions. First we define the maximally similar worlds to the actual one where A is true, $max(i,A)$. Lewis takes the maximally similar worlds as the limit of the evaluation universe. Instead of $max(i,A)$, for each operator $\rightarrow^{x}$ the limit is now the worlds as similar to i as max(i,A) from the point of view exclusively of the designed set X_i (and the designed sets more important than X_i), not taking into account least important strata.

(DEF 5.1). Given a world i K, a comparative similarity relation $\prec_i$ and a sentence then
$max(i,A) = \{l \quad [A] \mid \neg \ (m \quad K)(m \quad [A] \quad \neg(l \prec_i m))\}$

(DEF. 5.2.) For every x N and i K, i $[A \rightarrow^{x} B]$ iff either
i) [A] = , or
ii) [A] ≠ and T [A→B], where
$T = \{l \quad K \mid \quad m \quad max(i,A) \quad y \quad N \neg(\ x<y) \rightarrow (c_i^{y}(l) \leq c_i^{y}(m)))\}$

Let us take the physical implication “$A \rightarrow^{p} B$” as an example. I would call a world k physically accessible from i if and only if $c_k^{n} = c_k^{p} = 0$. I think that this notion is identical to Burks’s one. In the same way I will call a sentence A physically possible in k if an only if A holds at some world k physically accessible from i (otherwise A is physically impossible).Take a conditional statement $A \rightarrow^{p} B$. If A is logically impossible, then there is not world k such that A is true at k, and “$A \rightarrow^{p} B$” is trivially true by the first part of the definition. If A is physically possible then the evaluation universe is the set of physically possible worlds: c_i^{k} (max(i,A)) must be zero (because A is true at least in one possible world). If A is physically impossible, then the evaluation universe is the set of all the worlds as similar to i from the exclusive point of view of its physics as the most similar worlds to i where A is true. That is to say, in order to evaluate the conditional we change the physics of our world as much as it is required to make A true, but then we must allow the accidental and purely contingent sentences change freely, so we assure that the truth or falseness of the consequent does not depend on them. In this way, example (3.2) is modelled with $\rightarrow^{p}$ like example (2.2): it applies to a variety of situation (represented by the worlds of the evaluation universe), and its truth depends on the physical structure of the world.

Operators $\rightarrow^{n}$, $\rightarrow^{p}$, $\rightarrow^{a}$ have the same syntactic properties, their logic is Lewis' VW. The logic for $\rightarrow^{c}$ is VC conditional logic of Lewis, whose peculiarity is the validity of C: A B $\rightarrow$ (A$\rightarrow^{c}$B). This follows from exhaustivity and point (K.2) in definition 4.1. An important rule is the rule of impliciteness, which determines the entailment relations between operators:

(R.I) For every x,y N, if x<y then from A$\rightarrow^{x}$B conclude A$\rightarrow^{y}$B[14].

I will give now an informal and very brief description of each of the conditional operators.

The contingent implication ("$\rightarrow^{c}$") is equivalent to Lewis operator. It is the weakest of the operators. This weakness shows itself in CS. By the definition of the similarity relation (and the requisite (3) of the structuring function) no world *k* is as similar to another world *i* as *i* itself, so if the antecedent is true in *k* no other worlds need to be examined to know the truth value of the conditional statement, and the axiom CV becomes valid. One important feature about the conditional statements that the contingent implication tries to model is that the antecedent being contingent itself is not required. Consider, for example, the situation represented in the picture below:

and the statement:

(5.1) If triangles are parallelograms, then there is a parallelogram on the left of the circle.

Its formal translation is "A$\rightarrow^{c}$B", and the statement becomes true. We change the semantic structure of the actual world (the set N_i) to make A true, and arrive to max(*i*,A). Then we take as the upper bound of the evaluation universe the worlds as similar to the actual one as max(*i*,A) according to all the criteria of similarity, an we find that in these worlds the triangle is still on the left of the circle.

The actual implication ("$\rightarrow^{a}$") correspond to the relation between the antecedent and consequent of the statement (3.4). It is a sort of in-

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[14] See Vilanova [22] for a full exposition of the logics and the corresponding proofs.

termediate between the contingent and the physical implication. It applies to a variety of situations, not only the most similar to the actual, but all the situations which shares the conceptual framework, the physics and also the structural qualities of this world. I think that Åquist [1] and Nute [14] operators correspond more or less to the actual implication of mine. But while they think that theirs is the only valid one for the subjunctive conditional I consider that contingent implication is required for some kinds of subjunctive conditionals, as for example (3.3). In this way, the introduction of the actual implication put an end to the polemic between the paradigms of maximal and sufficient revision, whose battlefield was CS[15]. CS is valid for a class of counterfactuals (those strong counterfactuals modelled with $\rightarrow^{a}$) but invalid for another class (weak counterfactuals modelled with $\rightarrow^{c}$).

The physical implication ("$\rightarrow^{p}$") applies to the biggest part of the indicative conditional statements (excepting analytical conditionals). Nevertheless the differences (which are a lot), Burks' operator as well as Fetzer & Nute's are equivalent to mine. But $\rightarrow^{p}$ also applies to a special kind of subjunctive conditionals, what I have called "substantive counterfactuals" in the second paragraph. The inclusion of the substantive counterfactuals into the set of physical implications is, from my point of view, a important step towards the understanding of the real scientific discourse. Science can also speaks in subjunctive: it can talk about what would occur if some physical principle were violated, or investigate the consequences of a theory which is still not contrasted or is false, examine hypotheses, look for counterarguments to rival theories, etc. All these kinds of scientific argumentation require the use of substantive counterfactuals[16].

The semantic or analytical implication ("$\rightarrow^{n}$") applies to conditionals of the kind "If you are a bachelor, you are a man", or "If John is taller than Peter, Peter is shorter than John". These kind of statements are usually known as analytical truths. Some authors put them among the logical truths, but I cannot agree. The difference is that a logical falseness implies logically anything, but a analytical falseness does not imply analytically everything. Take those two sentences:

(5.2) If bachelors were married men, then there would be single married people.

(5.3) If bachelor were married men, then bachelor were red spirits.

[15] See Nute [14] for an explanation of the rivalry between the two paradigms.
[16] See Vilanova [20] for a discussion about this topic.

Example (5.2) is true, but (5.3) is false. This shows that even when we put in suspense our semantic notions, we are able to reason what follows and what does not.

6. Indicative and subjunctive revisited

We have seen that the difference between the conditional types is on the implicational strength. The indicative or subjunctive mood does not express different kinds of conditional relation. There is not “special operators” which pick up these differences of verbal mood. Statements (2.2) and (3.2) are modelled with the same operator ($\rightarrow^{p}$). The same conditional relation is expressed by statement (3.1) when it is the fact that Peter has eaten salt in the moment of the utterance and when it is not. This does not mean that the distinction indicative/subjunctive has not logical relevance, but that it means the same in the context of conditional statements as into other statements. It is important to note that the “subjunctiveness” or “indicativeness” is not a property of the whole conditional statement, but only of the antecedent and, more concretely, of the antecedent main verb. Let us investigate, then, what the subjunctive mood express about the antecedent.

To start with, let us take a look to what linguistics says about it. According to the linguistic account, in subjunctive mood we speak of a “hypothetical”, “fictitious” case, of something not-real, while in indicative mood we speak of a “de facto” case. From the logical point of view, “hypothetical” may mean various things. In order to explain this point, I will introduce some modal notions in my system. Let us define four necessity and possibility operators, each of them corresponding to one of the conditional operators:

DEF. 6.1: For any x   N,
$\Box^{x}A =_{df} (A \quad \neg A) \rightarrow^{x} A$
$\Diamond^{n}A =_{df} \neg\Box^{x}\neg A.$

It is easy to prove that for every world i, i [$\Box^{x}$A] iff for every world k such that $c_i^{x}=0$, k [A]. In other words, $\Box^{x}$A is true in i if A is true in every x-lly accessible from i world[17]. These definitions may pro-

[17] It is possible to give a precise definition of the term “x-ly accessible world”. A word k is analytically accessible from i iff $c_i^{n}(k) = 0$. k is physically accessible from i iff $c_i^{n}(k) = c_i^{p} = 0$. k is actually accessible from i iff $c_i^{n}(k) = c_i^{p}(k) = c_i^{a}(k) = 0$. k is contingently accessible from i iff $c_i^{n}(k) = c_i^{p}(k) = c_i^{a}(k) = c_i^{c}(k) = 0$. The unique world contingently accessible from i is, of course, i itself.

vide five different senses of "fictitious": logically impossible ($\neg\Diamond$), analytically impossible ($\neg\Diamond^{n}$), physically impossible ($\neg\Diamond^{p}$), actually impossible ($\neg\Diamond^{a}$) and contingently impossible or simply false ($\neg\Diamond^{c}$, or $\neg$). Generally, the sense of fictitious expressed by the subjunctive mood is the same type that the conditional relation of the whole statement. I will provide five examples, together with their formal translation:

(6.1) If the door were open, then the wind would blow $(\neg\Diamond^{c}A \quad (A \rightarrow^{c} B))$

(6.2) If Madrid were not the capital of Spain, then it would be easier to drive there $(\neg\Diamond^{a}A \quad (A \rightarrow^{a} B))$.

(6.3) If the water boiling point were 30 degrees, tropical lakes would evaporate $(\neg\Diamond^{p}A \quad (A \rightarrow^{p} B))$.

(6.4) If humans were not animals, humans would not be living things $(\neg\Diamond^{n}A \quad (A \rightarrow^{n} B))$.

(6.5) If it is the case that A and $\neg$A, then everything is the case $(\neg\Diamond A \quad (A \quad B))$.

Anyway, I will not define the corresponding "subjunctive implications" for two reasons. On the one hand, I do not see the point of defining special operators when subjunctives statements can be modelled without them and the subjunctive property belong to the antecedent. On the other hand, the coincidence of the type of "fictitious" and the type of the implication is just a regularity, and not a rigid rule. The type of "fictitious" depends more on the speech context and the conversational presuppositions. What we consider as "fictitious" at some moment may be considered "realistic" at other moment. This type of ambiguity is not unlike the ambiguity that affect to the modal idioms or the preposition "or" (inclusive, exclusive).

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