Logica Trianguli, 2, 1998, 3-16

How to Mix Alethic, Deontic, Temporal, Individual Modalities

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Abstract
Deontic logic handles not only deontic modalities, but also alethic and temporal ones. In addition, individuals like *authorities* and *addressees* play an important role. R5-D5 is a system handling alethic, deontic and temporal modalities, whose adequacy (i.e. soundness plus completeness) has been proved in an earlier paper. Similarly for KD*UXY with sets of individuals (without a proof of completeness). The present article is an attempt to construct a general system mixing R5-D5 and KD*UXY.

1. Introduction

In [4] I constructed canonical models for a temporal deontic system I named R5-D5. In this system, the real world is related to alternative *histories* (*paths*), among which some may be "good" and only one (not good) can represent the real history of our world. Some improvements can be made into [4], which I will not consider here. I will only remark that a (not serious) problem arises from the fact that on the one hand the proof of completeness of [4] permits us to draw a ramified structure from the axiomatic of R5-D5, and on the other hand it is known that there is no axiomatization of ramified structures. The solution, I think, consists in the sort of graphics the structure is intended to be interpreted; if a branching graphics is used, then everything is "normal"; if not, the graphic interpretation looks rather strange, but is acceptable, too[1].

Another important issue in deontic logic is that of individuals playing a role in norms, mainly their *authorities* and *addressees*. Again in previous works (mainly [2]), I constructed logical systems which assumed that there is a complete compatibility between the norms emitted

[1] Another question concerns the status of the set of "instants". One may suppose here that it is the set of real numbers. I will not study the problems connected to this.

by different authorities, which is, I confess, rather an idealistic view. Thus, for example, if *x* makes it obligatory that *p*, *y* makes it obligatory that *q*, then whatever *z* must make it permitted that *p* and *q*. I also showed that it is almost indispensable to use *sets* of authorities. In the example, since *p* is made obligatory (by *x*) and *q* (by *y*), *p* &*q* is made obligatory. But by whom? By their set, of course. The situation was similar concerning addressees and the result of this type of analysis was a system KD*UXY (vide *infra*) which captures the main intuitions at issue.

The problem now is to mix alethic, deontic, temporal modalities (R5-D5) and deontic individuals (KD*UXY) into a complete system as rational as possible.

To begin with, I will briefly recall what are the main features of R5-D5 and of KD*UXY. Then I will present the whole operation of mixing according to the two possible implementations, extending R5-D5 by adding individuals, or extending KD*UXY by adding time[2].

2. A preliminary combination of alethic, deontic and temporal modalities: R5-D5

We consider

R_t A = it is realized at the instant *t* that A [3]

$\Box$A = it is necessary that A

*O*A = it is obligatory that A

2.1. Axiomatics

R_t complies with the system $\mathbb{R}$:

Kt	$R_t(A \rightarrow B) \rightarrow (R_t A \rightarrow R_t B)$
D!t	$R_t A \leftrightarrow \neg R_t \neg A$ [4]
Rtt'	$R_{t'} R_t A \leftrightarrow R_t A$
R**N**t	A / R_t A [5]

[2] And alethic modalities (but this step presents no difficulties).

[3] To be precise, R_t is a family of operators indexed by *t*.

[4] See [6], p. 93. From a general point of view, the present notation of axioms and rules follows Chellas' book of 1980.

[5] The notation A / B means that if A is a thesis then B is a thesis, too. As it is known, other ways of writing this are $\vdash A \rightarrow \vdash B$, $\frac{A}{B}$. In my previous works appeared also the rule

similarly □ with the system S5

K□	$\Box(A \to B) \to (\Box A \to \Box B)$
T□	$\Box A \to A$
E□	$\neg\Box A \to \Box\neg\Box A$
R**N**□	A / □A

and *O* with KDU[6]

K*O*	$O(A \to B) \to (OA \to OB)$
D*O*	$OA \to \neg O\neg A$
U*O*	$O(OA \to A)$
R**N***O*	A / *O*A

In addition, R_t, □ and *O* verify some axioms proper to their combination:

□t1	$R_t\Box A \leftrightarrow R_t\Box R_t A$
□t2	$t' \leq t \quad R_{t'}\Box R_t A \to R_t\Box R_t A$
*O*t1	$R_t OA \leftrightarrow R_t OR_t A$
*O*t2	$t' < t \quad R_{t'} O(R_{t'} OR_t A \to R_t OR_t A)$
□*O*	$\Box A \to OA$
□*O***4**	$OA \to \Box OA$
□*O*t	$t' < t \quad R_{t'} OR_t (OA \to \Box A)$[7]

2.2. Semantics

Formulas are evaluated on branching frames built on two triadic relations, R and S. The following rules, among others, hold for the valuation function:

$\models_{\alpha,t} R_{t'} A$ iff $\models_{\alpha,t'} A$[8]

$\models_{\alpha,t} \Box A$ iff for every β such that Rtαβ, $\models_{\beta,t} A$

RN$_c$t $\quad R_t A$ / A $\quad$ (t not free in A)

which plays a role in [8]. I am not sure now that it is necessary, at least if no specification is made about the set of instants.

[6] KDU is Chellas' terminology, too.

[7] See [4] or [5] for more details about the axiomatics of R5-D5.

[8] More exactly, the evaluation of $R_{t'}$A primarily rests on a relation $T_{t'}$ on worlds (i.e. states of paths at some instant). But the axiomatics for $R_{t'}$ permits to replace the mecanism of evaluation with this relation by the present one. For more details, see [4].

$\models_{\alpha,t}$ OA iff for every β such that Stαβ, $\models_{\beta,t}$ A

where '$\models_{\alpha,t}$' [9] means "it is true on the path α at time t that…", 'Rtαβ' "β is an alternative to α from time t", 'Stαβ' "β is a good alternative to α from time t".

I will restrict myself to present here some semantic properties not as common as reflexivity or euclidianism (t is presupposed in front of all properties):

— St-secondary-reflexivity: α β (Stαβ Stββ)
— Rt-ramification: t' α β [(t'<t & R tαβ) R t'αβ]
— St-secondary-ramification:
t' α β γ [(t'<t & S t'αβ & S tβγ) St' βγ]
— SRt-implication: α β (Stαβ Rtαβ)
— RSt-transitivity: α β γ [(R tαβ & S tβγ) Stαγ]
— RSt-post-implication:
t' α β γ [(t'<t & S t'αβ & R tβγ) Stβγ][10]

2.3. Adequacy
Here is the table of adequacy (i.e. soundness and completeness). Proofs were given in [4].

Kt of $\mathbb{R}$	standardness of the model
D!t of $\mathbb{R}$	functionality of T_t
Rtt' of $\mathbb{R}$	uniqueness of path for evaluation of R_t
R**N**t of $\mathbb{R}$	standardness of the model
K□ of S5	standardness of the model
T□ of S5	Rt-reflexivity
E□ of S5	Rt-euclidianism
R**N**□ of S5	standardness of the model
K*O* of KDU	standardness of the model

[9] In the sequel I will write only '$\models_{\alpha,t}$' instead of '$\models^{\mathcal{M}}_{\alpha,t}$' (to mean it is true, in the model $\mathcal{M}$, on the path α at time t that).

[10] Again see [4] or [5] for more details about the semantics of R5-D5.

D*O* of KDU	St-seriality
U*O* of KDU	St-secondary-reflexivity
R**N***O* of KDU	standardness of the model
☐t1	$t' = t$ in $\beta_{t'}$ st α_t R $\beta_{t'}$ (see [4])
☐t2	Rt-ramification
*O*t1	$t' = t$ in $\beta_{t'}$ st α_t S $\beta_{t'}$ (see [4])
*O*t2	St-secondary-ramification
☐*O*	SRt-implication
☐*O***4**	RSt-transitivity
☐*O*t	RSt-post-implication

3. A deontic system with sets of authorities and addressees: KD*UXY[11]

3.1. Axiomatics

KD*UXY is as KDU, except that it adds sets of authorities (*X*, *X'*,…) and sets of addressees (*Y*, *Y'*, …) as arguments, or indexes, of operators and introduces two sorts of normative operators:

Let E = {*x*,*x'*,…} be a finite set of individuals (authorities and/or addressees) and let *X*, *X'*,… be various subsets of E. We define

O_{XY}A : (some subset of) *X* makes it obligatory for (every subset of) *Y* that A

Def. P_{XY} P_{XY}A = ¬O_{XY}¬A, and therefore

P_{XY}A : (every subset of) *X* permits for (some subset of) *Y* that A

O'_{XY}A : (some subset of) *X* makes it obligatory for (some subset of) *Y* that A[12]

Def. P'_{XY} P'_{XY}A = ¬O'_{XY}¬A, and therefore

P'_{XY}A : (every subset of) *X* permits for (every subset of) *Y* that A

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[11] The star after 'D' only recalls that the properties of X and Y are not exactly the same and there are two types of norms. Another notation could be KDD'UXY.

[12] In order to avoid confusion between the properties of *O* and *O'*, remember that *O'* has the same structure with respect to authorities and addressees: only *some* subset of them are present.

Clearly the prime norms are needed by the permission P'_{XY}, which is more intuitive than P_{XY} since a permission usually concerns every member of a set of human beings.

Thus KD*UXY is the set of the following axiomatic elements[13] :

$$
\begin{array}{ll}
\mathbf{K}O^*_{XY} & O^*_{XY}(\mathrm{A} \rightarrow \mathrm{B}) \rightarrow (O^*_{XY}\mathrm{A} \rightarrow O^*_{XY}\mathrm{B}) \\
OO'_{XY} & O_{XY}\mathrm{A} \rightarrow O'_{XY}\mathrm{A} \\
\mathbf{D}O'_{XY} & O'_{XY}\mathrm{A} \rightarrow \neg O'_{XY}\neg \mathrm{A} \\
\mathbf{U}O^*_{XY} & O^*_{XY}(O^*_{XY}\mathrm{A} \rightarrow \mathrm{A}) \\
\mathrm{R}\mathbf{N}O_{XY} & \mathrm{A} \;/\; O_{XY}\mathrm{A} \\
\mathrm{R}\mathbf{Sr}O_{XY}\ (X \quad U \,\&\, Y \quad V) & O_{XV}\mathrm{A} \;/\; O_{UY}\mathrm{A} \\
\mathrm{R}\mathbf{Sr}O'_{XY}\ (X \quad U \,\&\, Y \quad V) & O'_{XY}\mathrm{A} \;/\; O'_{UV}\mathrm{A}
\end{array}
$$

Most of these elements are simple extensions of those of KDU[14]. The OO'_{XY} axiom is self intuitive. Only the last two rules have to be explained. The former, R**Sr**O_{XY}, states that: (i) if one set of authorities is included in another, then every obligation emanating from the former is also an obligation emanating from the latter and (ii) if one set of addressees *includes* another one, then every obligation for the former implies obligation for the latter (remember that O_{XY} is an obligation for *every* subset of Y). Of course this rule includes an idealistic general normative compatibility between different authorities[15]. As for the last rule, the reverse order of the inclusion between sets of addressees obviously follows from the fact that O'_{XY} is an obligation for *some* subset of Y.

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[13] This time, the star '*' after O means that the axiom is valid for O and O'. KD*UXY is DBXY of [2], p. 469 and [3], p. 105, *plus* **U***O_{XY}.

[14] OO'_{XY} and **D**O'_{XY} implies $O_{XY}\mathrm{A} \rightarrow \neg O_{XY}\neg\mathrm{A}$. So no **D**$O_{XY}$ has to be laid down. The same for R**N**O'_{XY}. On the contrary, **U**O_{XY} and **U**O'_{XY} are necessary, since neither is derivable from the other.

[15] For more details, see [2], p. 469.

3.2. Semantics[16]

There are two relations, S and S', respectively for O_{XY} and O'_{XY}, possibly defined as

$$SXY\,ww' = \underset{i}{\&}\ \underset{j}{\bigvee}\ Sx_i\,y_j\,ww'$$ [17]

$$S'XY\,ww' = \underset{i}{\&}\ \underset{j}{\&}\ Sx_i\,y_j\,ww'$$

where the primitive relation $Sx_i\,y_j\,ww'$ holds between individuals (authorities x_i and addressees y_j) and two worlds.

Then the following conditions obtain:

— S'SXY-implication (i.e. inclusion of S' into S):

$S'XY\,ww' \quad SXY\,ww'$

— S'XY-seriality, and therefore SXY-seriality (by S'SXY-implication), i.e.

$w \quad w' \quad XY\ S'XYww'$

— S*XY-secondary-reflexivity: $S^*XY\,ww' \quad S^*XY\,w'w'$[18]

— SXY-set-semicoregularity:

$(X \quad U\ \&\ Y \quad V) \quad (\ SUY\,ww' \quad SXV\,ww'\ )$

— S'XY-set-regularity:

$(X \quad U\ \&\ Y \quad V) \quad (\ S'UV\,ww' \quad S'XY\,ww'\ )$

3.3. Adequacy

The proof of soundness is a routine task; it suffices to show that all axioms are valid and rules preserve validity. For completeness, I suggest that it be proved using canonical models along the lines of [6], as in [4] for R5-D5. I will restrict myself here to exhibit the correspondence between semantic properties and axiomatic elements:

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[16] Vide [5] for a more intuitive and extended presentation of the semantics for KD*UXY, particularly on construing primitive semantics handling *individuals*, not sets of them.

[17] It is a delicate problem to know whether $\underset{i}{\&}\ \underset{j}{\bigvee}\ Sx_i\,y_j\,ww'$ or $\underset{i}{\bigvee}\ \underset{j}{\&}\ Sx_i\,y_j\,ww'$ should be written. No special hypothesis, as far as I can see, permits to choose the former or the latter. In any case, it does not matter for our present study since all the same set-semantic properties can be derived from the individual ones, according to either definition.

[18] The star '*' means that the property holds for S and S'.

KO^*_{XY}	standardness of the model for O^*
OO'_{XY}	S'SXY-implication
DO'_{XY}	S'XY-seriality
UO^*_{XY}	S*XY-secondary-reflexivity
R**N**O_{XY}	standardness of the model for O
R**Sr**O_{XY}	SXY-set-semicoregularity
R**Sr**O'_{XY}	S'XY-set-regularity

4. Mixing R5-D5 and KD*UXY: R5-D*XY5

The general principle of mixing can be summarized in the following table, which treats the problem semantically:

R5-D5 →	**R5-D*XY5**	← **KD*UXY**
□, O, R_t	□, O_{XY}, O'_{XY}, R_t	O_{XY}, O'_{XY}
	S'StXY-implication	S'SXY-implication
St-seriality	S'tXY-seriality	S'XY-seriality
St-secondary-refl.	S^*tXY-secondary-refl.	SXY-secondary-refl.
St-secondary-ram.	S^*tXY-secondary-ram.	
property for Ot1	property for O^*_{XY}t1	
SRt-implication	SRtXY-implication	
RSt-transitivity	RStXY-transitivity	
RSt-post-implic.	RStXY-post-implic.	
	StXY-set-semicoregul.	SXY-set-semicoregul.
	S'tXY-set-regularity	S'XY-set-regularity

There are two ways to implement the mixing (see the arrows at the top of the table). One can
- either pass from R5-D5 to R5-D*XY5 by changing the norm *O* into O_{XY} and O'_{XY}, and expanding its semantic properties;
- or pass from KD*UXY to R5-D*XY5 by temporalizing the semantic properties which correspond to the norms O_{XY} and O'_{XY}.

Most cases are obvious. For example, the axiom[19]

$\Box O \qquad \Box A \rightarrow OA$,

corresponding to the semantic property of SRt-implication (S*t* αβ R *t* αβ) must be converted into

$\Box O_{XY} \qquad \Box A \rightarrow O_{XY}A$

corresponding to SRtXY-implication:

S*tXY* αβ R *t* αβ

So, everything which is necessary belongs to the things which are good according to any *X* and *Y*.[20]

Thus, semantically, the alethic relation, R (of R5-D5), must remain unchanged, while the synthesis of S *t* αβ[21] (of R5-D5), S*XY*ww' and S'*XY*ww' (of KD*UXY) leads to two relations with five arguments, S*t XY*αβ and S' *t XY*αβ (of R5-D*XY5), both of which can be read "From time *t*, the path β is a variant of the path α, which is good according to the set of authorities *X* and the set of addressees *Y*" (the sole difference in the reading of S and S' is attached to the meaning of "set of addressees"). By this approach, one reaches the following semantic properties without any particular problem:

— S'StXY-implication: S' *tXY* αβ S *tXY* αβ
— S'tXY-seriality, and therefore StXY-seriality (by 1), i.e.
  α β *XY* S *t XY*αβ
— S*tXY-secondary-reflexivity: S* *t XY*αβ S* *t XY*ββ
— StXY-set-semicoregularity:
  (*X* *U* & *Y* *V*) (S*tUY* αβ S*tXV* αβ)

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[19] See [2] p. 291, [3] p. 54 for a general discussion about this axiom.

[20] Notice that we need to lay down neither $\Box O'$ nor S'RtXY-implication, which can be derived.

[21] Because I am not explicitely working with canonical models in the present paper, I prefer the formulation 'R *t* αβ' and 'S *t* αβ' instead of 'α_t R β_t' and 'α_t S β_t', used in [4].

— S'tXY-set-regularity:

$(X \quad U \;\&\; Y \quad V) \qquad (\, \mathrm{S}tUV\alpha\beta \qquad \mathrm{S}tXY\alpha\beta\,)$

These properties are respectively "adequate" to the axioms and rules:

$OO'_{XY} \qquad O_{XY}\mathrm{A} \;\rightarrow\; O'_{XY}\mathrm{A}$

$\mathbf{D}O'_{XY} \qquad O'_{XY}\mathrm{A} \;\rightarrow\; \neg O'_{XY}\neg\mathrm{A}$

$\mathbf{U}O^*_{XY} \qquad O^*_{XY}(O^*_{XY}\mathrm{A} \;\rightarrow\; \mathrm{A})$

$\mathrm{R}\mathbf{Sr}O_{XY} \quad (X \quad U \;\&\; Y \quad V) \qquad O_{XV}\mathrm{A} \,/\, O_{UY}\mathrm{A}$

$\mathrm{R}\mathbf{Sr}O'_{XY} \quad (X \quad U \;\&\; Y \quad V) \qquad O'_{XY}\mathrm{A} \,/\, O'_{UV}\mathrm{A}$

From the axiomatic viewpoint it should be added the "temporalized" versions of $\mathbf{K}O_{XY}$, $\mathbf{K}O'_{XY}$ and $\mathrm{R}\mathbf{N}O_{XY}$, which are semantically illustrated only by the "standardness" of the model (i.e. there corresponds no property at all to them).

Finally the sole non trivial case is that of S*tXY-secondary-ramification

$(t'\!<\!t \;\&\; \mathrm{S}^*\, t'XY\alpha\beta \;\&\; \mathrm{S}^*\, tXY\beta\gamma) \qquad \mathrm{S}^*\, t'\, XY\beta\gamma$

with its axiom O^*_{XY}t2

$t' < t \qquad R_{t'}\, O^*_{XY}(R_{t'}\, O^*_{XY}R_t\, \mathrm{A} \;\rightarrow\; R_t\, O^*_{XY}R_t\, \mathrm{A})$

The problem is to know whether the same sets of individuals must be kept in the entire property (and axiom) as it appears here or they may be changed, to obtain e.g.: [22]

$(\mathrm{S}^*\, t'XY\alpha\beta \;\&\; \mathrm{S}^*\, tUV\beta\gamma) \qquad \mathrm{S}^*\, t'\, UV\beta\gamma$

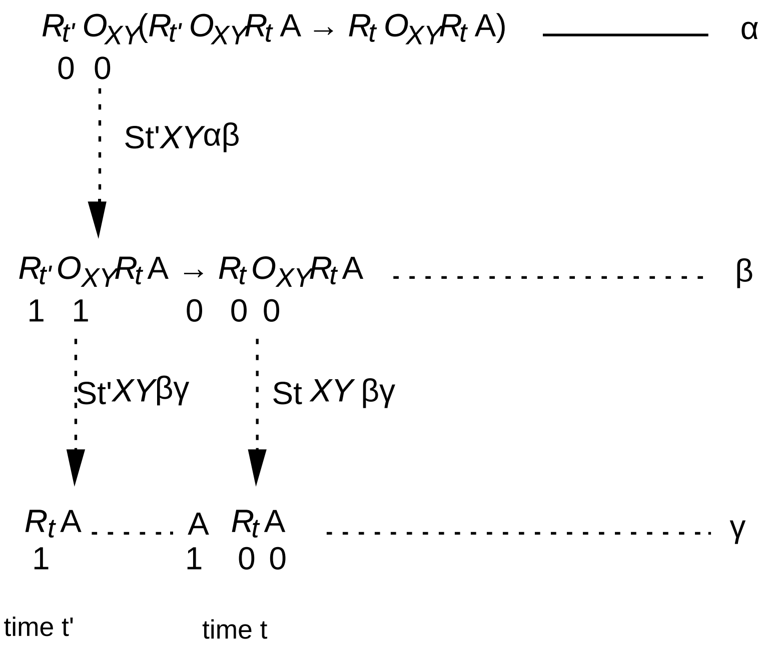
or

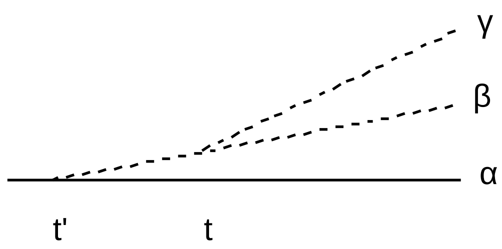
$(\mathrm{S}^*\, t'XY\alpha\beta \;\&\; \mathrm{S}^*\, tUV\beta\gamma) \qquad \mathrm{S}^*\, t'\, XY\beta\gamma$

For simplicity, I will consider only the "S-part", i.e. without the "S'-counterpart" (and therefore, axiomatically, the "*O*-part") of the case.

Let us draw the diagram of O_{XY}t2.

[22] From now on, I omit $t' < t$, which will be always supposed to be true.





β is a path good according to *X* and *Y* at time *t'*. The absurdity concluding the proof (A=1 and A=0 on γ at time *t*) stems from the fact that the conjunction of S*t' XY*αβ and S*t XY*βγ implies S*t' XY*βγ, i.e. the StXY-secondary-ramification. Of course, this last property is as intuitive as its correspondent of R5-D5, StXY-secondary-ramification. Thus there is no problem at this stage.

However, the problem is not so simple, because the property

(S*t' XY*αβ & S *t UV*βγ)   S*t' XY*βγ   (*)

can be derived in the semantics.

Proof

First, on the basis of RStXY-post-implication and SRtXY-implication (used with *UV* instead of *XY*), it is not difficult to derive

$$(\mathrm{S}\,t'\,XY\alpha\beta \;\&\; \mathrm{S}\,t\,UV\beta\gamma) \qquad \mathrm{S}\,t\,XY\beta\gamma \qquad (**)$$

Since RStXY-post-implication and SRtXY-implication appeared plausible, the property (**) should be admitted.
Then, (**) jointed to the StXY-secondary-ramification, immediately implies the desired result, (*), with *t'* instead of *t* in the consequent. ■

The corresponding thesis is:

$$R_{t'}\,O_{XY}(R_{t'}\,O_{XY}R_t\,\mathrm{A} \rightarrow R_t\,O_{UV}R_t\,\mathrm{A}) \qquad (\mathrm{T}^*)$$

On the contrary, it may also be shown that the analogous formula with *UV* two times,

$$R_{t'}\,O_{XY}(R_{t'}\,O_{UV}R_t\,\mathrm{A} \rightarrow R_t\,O_{UV}R_t\,\mathrm{A}) \qquad (\mathrm{T}^{***})$$

cannot be derived and the property

$$(\mathrm{S}\,t'\,XY\alpha\beta \;\&\; \mathrm{S}\,t\,UV\beta\gamma) \qquad \mathrm{S}\,t'\,UV\beta\gamma \qquad (***)$$

is not admissible.

The meaning of this is straightforward. The goodness according to *U* for *V* at time *t* viewed by *X* for *Y* from time *t'* is goodness according to *X* for *Y* at time *t* (cf. (**)) or even *t'* (cf. (**)), but not goodness according to *U* for *V* from time *t'*. In other words, deontic feedback of (*U*, *V*) onto (*X*, *Y*) is not conceivable. Is it not it a wise solution?

Perhaps there are other alternatives for the "XY-zation" of some axioms or theses of R5-D5. But the matter looks rather muddled. I will not go on in this direction and prefer to end the study by presenting a table of adequacy for R5-D*XY5 (the reader will have no difficulty to write explicitly axioms, rules and semantic properties).

$\mathbb{R}$ and S5 as for R5-D5 (see Section 2)	
KO^*_{XY} of KD*UXY	standardness of the model for O^*
OO'_{XY} of KD*UXY	S'StXY-implication
DO'_{XY} of KD*UXY	S'tXY-seriality
UO^*_{XY} of KD*UXY	S*tXY-secondary-reflexivity
R**N**O_{XY} of KD*UXY	standardness of the model for O
R**Sr**O_{XY}	StXY-set-semicoregularity
R**Sr**O'_{XY}	S'tXY-set-regularity
☐t1	$t' = t$ in $\beta_{t'}$ st $\alpha_t \mathrm{R} \beta_{t'}$ (see [4])
☐t2	Rt-ramification
O^*_{XY}t1	$t' = t$ in $\beta_{t'}$ st $\alpha_t \mathrm{S} \beta_{t'}$ (see [4])
O^*_{XY}t2	S^*tXY-secondary-ramification
☐O_{XY}	SRtXY-implication
☐O^*_{XY}**4**	RS^*tXY-transitivity
☐O^*_{XY}t	RS^*tXY-post-implication

6. Conclusion

As we remarked at the end of the previous section, there is some indetermination on which axioms (or semantic properties) should be laid down about cases with a plurality of sets of individuals. However, a more serious drawback concerns the construction of the system itself. Generally speaking, whenever a new modality is added to others in a system, this system is enriched with new features (axioms or rules, or semantic properties). Thus, mixing alethic and pure deontic modalities gives rise to both axioms ☐O (☐A → OA), ☐O4 (OA → ☐OA), i.e. SR-implication and RS-transitivity from the semantic point of view. Similarly for mixing *separately* R_t and ☐, R_t and O. Finally, when the three modalities R_t , ☐ and O appear together in the same system, we need ☐Ot ($t' < t$ $R_{t'} O R_t$ (OA → ☐A)), i.e. RSt-post-implication. On the other hand mixing sets of individuals into deontic logic yields the rules

$\mathrm{R}\mathbf{Sr}\,O_{XY} \quad (X \quad U \,\&\, Y \quad V) \qquad O_{XV}\mathrm{A} \,/\, O_{UY}\mathrm{A}$

$\mathrm{R}\mathbf{Sr}\,O'_{XY} \quad (X \quad U \,\&\, Y \quad V) \qquad O'_{XY}\mathrm{A} \,/\, O'_{UV}\mathrm{A}$

Now my question is: what new element is needed when we make the complete composition of alethic, deontic, temporal modalities with sets of authorities and addressees? I must confess that no new element should be apparently introduced.

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