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Analyticity, Justification and Self-Evidence

In his philosophical masterpiece *Die Grundlagen der Arithmetik* (*The Foundations of Arithmetic*) of 1884, Frege states his logicist project in terms of the notion of analyticity: A sound foundation of arithmetic (number theory and real analysis in the first place) must establish the analytic nature of its basic laws. At the outset of *Grundlagen* (in §3), Frege defines the notion of analyticity in terms of deductive proof: a truth is analytic if can be proved from primitive laws of logic (logical axioms) and admissible definitions. However, the definition is incomplete since it does not include the logical axioms and the definitions qua first premises of a proof (and the principles on which the admissibility of a definition rests). Moreover, before giving the definitions of „analytic" and also of „synthetic", „a priori" and „a posteriori" Frege emphasizes that the corresponding distinctions concern not the content of the judgement but rather the justification for making the judgement (*Urteilsfällung*). He adds that where the justification is missing the possibility of making the classification disappears. So, if we take both, these remarks and Frege's ensuing definition of „analytic truth" at face value, we seem to face a serious problem. Plainly, if the logical axioms and the definitions satisfying the criteria of admissibility were not considered to be analytic in their own right, that is, in a sense of „analytic" that is not explainable in terms of proof (= deductive justification), it would be unfathomable how the truths derived from them could enjoy the status of being analytic. There are a number of statements in Frege's work which leave no doubt that the truths derived by purely logical (and gapless) inference from logical axioms and admissible definitions inherit the epistemological status (logical or analytic) of the latter. And in my view this is highly plausible. In short, if „analytic" and „justification for making the judgement" are inseparably intertwined, as Frege claims, characterizing the logical axioms and admissible definitions as analytic, and thus extending or completing the definition of „analytic truth" by including logical axioms and definitions, requires „inexorably" (a) a non-deductive justification of acknowledging them as true and (b) an explanation of their analyticity which does not rest on the notion of deductive proof.

In the main part of my talk, I shall critically discuss these issues and comment also on the assumed analyticity of what in the literature on Frege has come to be known as „Hume's Principle" (HP). HP is the linchpin of his proofs of the basic laws of cardinal arithmetic. I conclude with the presentation of an epistemic dilemma and make a suggestion of how it might be resolved. According to Frege, an axiom of a theory T qua T-unprovable truth must be self-evident and at the same time possess real epistemic value. I argue that in the light of this conception Frege faces an epistemic dilemma whenever he intends to choose an abstraction principle of the form "$Q(a) = Q(b) = R_{eq}(a, b)$" as an axiom of a theory T. Here "Q" is a singular term-forming operator, a and b are free variables of the appropriate type, ranging over the members of a given domain, and "R_{eq}" is the sign for an equivalence relation holding between the values of a and b. The dilemma applies especially to Axiom V, but it would equally apply to Hume's Principle, if Frege were to select it as an axiom of T, or to the choice of any other abstraction principle: the case in which both real epistemic value and self-evidence are given their due is ruled out. I suggest that Frege might escape this epistemic impasse if of two evils he were to choose the lesser: replacing the notion of self-evidence as applied to axioms by a weaker epistemic notion while retaining the requirement of genuine knowledge or real epistemic value.