Tomasz Jarmużek

MASTER ARGUMENT VS. SEA-FIGHT TOMORROW

Abstract

This paper deals with the Aristotelian problem of Sea-fight tomorrow, but it is analyzed from a Diodorean point of view. Precisely, we examine whether the lost argument of Diodor Cronus, called Master Argument, could be conducted within a frame of future open time. Since we do not know the argument itself, we follow the known reconstructions of Master Argument. Each of them requires special logical assumptions, but also semantic constraints concerning a frame of time. The last detail interests us the most. Showing shortly, step by step, all constraints, we come to the conclusion that the strictly treated reconstructions of Master Argument must prefer a linear model of future.

1. Introduction

1.1. Aristotle and Diodor Cronus

In the famous Chapter IX of De Interpretatione Aristotle expressed his standpoint concerning the so called futura contingentia, i.e. propositions about contingent future events. His view supported indeterminism, since Aristotle thought that the future is open: the future seafight may happen or not. Omitting possible interpretations of the famous Aristotle’s fragment (like the three value interpretation of Jan Łukasiewicz), we could say

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that Aristotle was a proponent of branching time. He would probably believe that present and past are determined, but future consists of many unrealized possibilities, and some of them must exclude others.

Several years after Aristotle, Megarian philosopher and logician, Diodor Cronus invented an argument, called by his successors Master Argument. Unfortunately, the argument was lost but some important details are retained.

As ancient commentators claimed, Diodor had supported the standpoint that: something is possible iff it is or will be true and something is necessary iff it is and will always be true. In order to justify this view, Diodor assumed the negation of the first sentence and showed that it provides a contradiction. The premises of his reasoning were the following sentences:

\[(\text{D1}) \text{ Every proposition true about the past is necessary.} \]
\[(\text{D2}) \text{ An impossible proposition cannot follow from a possible one.} \]
\[(\text{D3}) \text{ There is a proposition which is possible, but which neither is nor will be true.} \]

Because Diodor thought that the first two premises were plausible, he denied the last one, obtaining his definition of the possibility. Using the Aristotle’s law: if it is not possible that it is not \( p \), then it is necessary that \( p \), he could also obtain the definition of the necessity. Taking as a subject of Diodorean modalities propositions or sentences, we have the Diodorean definitions of temporal modalities:

\[(D\Diamond) \text{ A sentence is possible iff it is or it will be true.} \]
\[(D\Box) \text{ A sentence is necessary iff it is and will always be true.} \]

Consequently, every reconstruction of Master Argument should assume premises (D1), (D2), (D3) on the ground of some logical system and look for inconsistency.

We know that Diodor was a determinist and fatalist. He supposed that the future (like the present and the past) were completely determined. Moreover, his notions of modalities seem deterministic themselves (which we analyze later shortly). Knowing the key details of the lost argument, currently some logicians and philosophers try to reconstruct it using modern tools of logic. Obviously, these reconstructions must deal with the representation of time included in the argument. Hence, those tools of logic are some kind of temporal or tense logic, and – as a consequence –
they must prefer a model of time, imposing constraints on a representation of time.

According to some historians and logicians, Diodor discussed the Aristotelian view on future consciously (see: Döring K., 1972, p. 134, or Prior A. N., 1962, s. 138). In this light we may expect that Master Argument presents the deterministic view on future.

1.2. Determinism and indeterminism

The notions of modalities which were supported by Diodor appear to be deterministic or fatalistic. We shall consider the following proposition:

(1) *I won a lottery.*

It appears that anyone who utters (1) makes a statement that expresses a possible state of affair. Even if he never wins the lottery (as we know the plausibility of such an event is very small), we still think that it is possible. Nevertheless, according the Diodorus’ definition (1) is possible if and only if I won a lottery or will have won. Hence, if I neither did nor will win a lottery, it is not possible that (1). But it contradicts our everyday view on the nature of possibility.

The Diodor’s notion seems to forbid many sentences to be possible. For example, we can easily think that the propositions:

(2) *Dobrawa is a chessmaster at the time* $t$.
(3) *Dobrawa is not a chessmaster at the time* $t$.

are both possible, but only one of them is true. According to the Diodorus’ notion only one of them is possible. So, for many ancient thinkers Diodorus supported the very deterministic concept of modality. It is clear when we take into account the truth-maker aspect of his definition. A state of affair is possible if and only if it takes place now or will take place in the future. Hence, the states of affairs that neither do nor will take a place in the history are impossible. At the same time every possibility must be realized and there are no unrealized possibilities. This is why ancient thinkers could have thought that Diodor’s concepts provide determinism or fatalism.

The deterministic view the history of the world can be illustrated as a line. Inversely, every indeterminist imagine the history of the world as a branching tree. In many points there are future possibilities. However, the past always is a line, the future has many variants. Therefore, following
Aristotle, we may say that although tomorrow one of the possibilities will be realized, today a tomorrow sea-fight may happen or not.

The above remarks indicate that time will be understood here not in a physical sense, but more generally, as a way of representing temporal possibilities.

1.3. Aim

We are going to present the results of some formal reconstructions of Master Argument. Especially, our aim is to show which assumptions about the nature of time appear in each reconstruction. The main problem is whether any of the known theories in which Master Argument is reconstructive has got a model of open, branching time, corresponding with the Aristotelian view. If it took place, we could say that Diodorean notions are not deterministic, and then even Diodor should agree that Sea-fight Tomorrow may still happen or not happen. Our conclusion is that none of the known reconstructions delivers a positive answer.

1.4. Time

For the sake of further considerations we shall introduce an usual conceptualization of time. Our approach to the problem is very simple, i.e. we shall treat time as if it consisted of points. This approach is common in modal or tense logic, where possible worlds are interpreted as dynamic states of the same world ordered in possible ways of its development.

However, we shall impose stronger conditions on the accessibility relation than it is usually done to make it more natural, better representing the usual concept of time. Moreover, in our considerations, it will have some common properties like transitivity and irreflexivity (hence, also asymmetry). We assume that the symbol “<” will denote a relation to be earlier than.

**Definition 1.1.** A time or a structure of time is an ordered set \((T, <)\), where \(T\) is a set of moments and \(<\) is an relation defined on \(T^2\) which satisfies the conditions:

i) \(\forall t_1, t_2, t_3 \in T (t_1 < t_3 \land t_2 < t_3 \rightarrow t_1 < t_2 \lor t_2 < t_1 \lor t_1 = t_2)\)

ii) \(\exists t_1 \in T \forall t_2 \in T (t_1 < t_2 \lor t_2 < t_1 \lor t_1 = t_2)\).
Using the above definition, we can define one more helpful notion:

**Definition 1.2.** A branch of time structure \( (T, <) \) is any nonempty subset \( T_1 \subseteq T \), such that:

i) \( \forall t_1, t_2 \in T_1 (t_1 < t_2 \lor t_2 < t_1 \lor t_1 = t_2) \),

ii) for any \( T_2 \subseteq T \) satisfying the condition i), if \( T_1 \subseteq T_2 \), then \( T_1 = T_2 \).

These notions guarantee us that none of time structures can be branched in the past, so every moment has to have, at most, one past, and for any branches there is at least one common moment. Simultaneously, the definition allows time to be branched in the future. The question is, if the tools that the Master Argument requires, allow for such a possibility.

2. Master Argument in positional logic

2.1. Systems of positional logic

In the article *Podstawy analizy metodologicznej kanonów Milla* (The Foundations of the methodological analysis of Mill’s canons, [7]) Jerzy Łoś proposed an operator \( R \) that refers sentences to temporal moments. Let us look, for example, at a sentence:

*It is raining in Łódź.*

From a logical point of view, it is a propositional function which does not have any logical value, unless we point at a temporal context from a fixed set of such contexts. If the sentence was considered today as a description of a state of affairs, it could be true. If it was considered yesterday, it could be false. The operator enables us to connect any sentence \( p \) with any temporal context \( t \). Such a built sentence we read as: a sentence \( p \) is realized at a temporal context \( t \) (a point of time, an interval of some kind, etc).

The operator of realization can be applied more widely than only to temporal contexts. One can find a review of these applications in [14]. In

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2Hence, as an indexical sentence, it expresses various logical propositions at different contexts of utterance, denotes different parts of a world, and can change a logical value.
the range of $R$ there may be also the so-called eternal sentences. There is no obstacle to think about a logical value of the sentence:

Polish-Lithuanian knights are winning The Battle of Grunwald on the 15th June 1410.

at such a temporal context as, for example, 1400. Maybe it was already true then, but maybe it had the third logical value?

Systems of $R$-logics are usually extended by quantifiers, neglecting $R$ operator, and additional operations. For example, we can number a domain and introduce adding and subtracting to measure a distance from one point to another. On the other hand, a set of positions can be ordered by some additional conditions having influence on a logical value of sentences. Here a lot of reasonable kinds of extension are available. A laconic review of possibilities can be found in the following papers: [13], [2], [5], [15].

A language of positional logics is made of Boolean connectives: $\neg$, $\land$, $\lor$, $\rightarrow$, $\leftrightarrow$, propositional sentences, the operator of realization: $R$, variables or/and constants for positions: $t_1, t_2, \ldots, a, b, c, \ldots$, quantifiers: $\exists, \forall$. If a metric structure of positions is assumed, the operations of adding (+) and subtracting (−) can be used, and we add variables/constants for distances: $d_1, d_2, \ldots$. Here we have some examples of formulas with their reading under temporal interpretation:

1. $R_{a}(p)$ – "$p$ is realized at time $a$"
2. $\exists t_1 R_{t_1}(p)$ – "there is a point of time at which $p$ is realized"
3. $\forall d_1 \exists t_1 (R_{t_1-d_1}(p) \rightarrow R_{t_1+d_1}(q))$ – "for any distance there is a point of time such that if $p$ is realized in some distance before that point, then $q$ is realized is the same distance after that point of time".

2.2. F. S. Michael’s approach to Master Argument

One of the interesting ideas is to interpret and reconstruct Master Argument in a metric propositional logic. This concept comes from F. S. Michael ([8]) who presents an outline of this approach. Unfortunately, it has got some drawbacks, one of them is an informal character of the presentation. For instance, Michael did not describe a logical system of reconstruction. This approach is improved by Jarmuzek T., ([3] book in preparation).

In that reconstruction we use only the premises (D1) and (D3). Obviously, in the language there must appear symbols for modalities, i.e. modal,
propositional operators □ and ♦. Moreover, we need a constant n for a present point of time, the point at which we evaluate sentences. In the metric propositional logic the Diodorean premises look like:

- (D1) \( R_{n-d}(A) \rightarrow □A \) – "if sentence A was realized in the past, then it is necessary that A"
- (D3) \( ♦p_0 \land \neg R_n(p_0) \land \neg R_{n+d}(p_0) \) – "for some sentence p_0: it is possible that p_0, but p_0 is not realized now and will never be realized”.

In the reconstruction apart from (D1) and (D3) we use some more premises that somehow correspond to the widely known modal principles:

- □A → R_n(A)
- ∃t R_t(A) → ♦A.

Finally, those premises allow to come to a contradiction. But the key point of the argument uses the following principle:

\( \exists t_1 \forall d_1 (R_{t_1-d_1}(A) \land R_{t_1}(A) \land R_{t_1+d_1}(A)) \rightarrow \forall t_2 R_{t_2}(A). \)

This axiom tells us that the order of positions must be linear, which is not strange, since in metric operations the coordinates of points must be assigned uniquely. Therefore, in this reconstruction the time is not open in the Aristotelian sense.

### 2.3. N. Rescher’s reconstruction

The next formalization of Master Argument is also expressed in a positional logic, but not metric one. The author, Nicholas Rescher, uses additional symbol "<" for an ordering relation on a set of temporal positions. The complete argument with a proper introduction can be found in the papers: [12], [14], [4]. It is worth saying that Rescher introduces modalities with indexes so, for instance, something is possible (necessary) at \( t_1 \) (\( ♦_{t_1}, □_{t_1} \)), but at an other time may already not be possible (necessary). Here we present only the result and Diodorean premises.

- (D1) \( R_{t_1}(A) \land t_1 < t_2 \rightarrow □_{t_2}(A) \)
- (D2) \( (♦_{t_1}(A) \land t_1 < t_2) \rightarrow ♦_{t_2}(A) \)
- (D3) \( ♦_n(p_0) \land (n < t_1 \lor n = t_1 \rightarrow \neg R_{t_1}(p_0)). \)
In Rescher’s approach contradiction is also obtained. But we must explicite assume that $<$ is a linear relation: asymmetric, transitive and connected: $t_1 < t_2 \lor t_1 = t_2 \lor t_2 < t_1$, for any $t_1$, $t_2$. Therefore, the concept of time in Rescher’s reconstruction excludes branching at all.

3. Master Argument in tense logic: Prior’s approach

Another way of representing time in argumentation is to use modal-tense language. The modal-tense language is built with Boolean language by adding tense: $F$, $P$, $H$, $G$ and modal operators: $\diamond$, $\Box$. For the first time it was applied for Master Argument reconstruction by A. N. Prior. The basic ideas were published in [9], [10]. The reconstruction was thoroughly examined from a logical point of view in [6].

Prior formalized Diodor’s premises as follows:

- (D1) $PA \rightarrow \Box PA$
- (D2) $\Box (A \rightarrow B) \rightarrow (\diamond A \rightarrow \diamond B)$
- (D3) $\diamond p_0 \land \neg p_0 \land \neg Fp_0$.

Also minimal, normal logic $K_t$ and two additional axioms in Prior reconstruction were involved:

- any tautology of transitive frames, for example: $FFA \rightarrow FA$
- (P) $A \land GA \rightarrow PGP$.

The formula (P) is not as innocent as it seems at first glance T. Jarmužek, A. Pietruszczak in [6] proved that it is a tautology of special kind of frames, defined below. Let $(T, R)$ be a frame. A relation of immediate successor $R' \subseteq R$ is determined by condition: $t_1 R' t_2$ wtw $t_1 R t_2$ and $\neg \exists t_3 (t_1 R t_3 \land t_3 R t_2)$.

**Definition 3.1.** [P-frame] We say that $(T, R)$ is a P-frame iff for any $t_1$, $t_2$, $t_3$:

1. if $\neg t_1 R t_1$, then $\exists t_2 t_2 R' t_1$
2. if $t_1 R' t_2$ & $t_1 R t_3$ & $t_2 \neq t_3$, then $t_2 R t_3$.

In [6] we showed and pictured branching structures of time that satisfy formula (P) and are transitive, so they are good interpretations for
Diodorus’ notions under Prior’s reconstruction. Frankly speaking, the solution is not satisfying, since in a standard tense logic we assume a weak interpretation of tense operator. It means, for example, that $F_p$ is true at $t$ if there is a $t'$ accessible from $t$ and $p$ is true at $t'$. But this interpretation is far from the natural reading. “It will be that case, that $p$” means on each possible branch at some point $t$, $p$ is true. The former interpretation – called weak – is better for reading $F_p$ as “It may be in the future the case that $p$”. Under the natural reading – called strong interpretation – a tautology of $K_t$: $A \rightarrow HFA$ excludes a branching time, since it introduces linearity, and consequently a future is closed and unique.

Another variant of Priorean reconstruction can be found in M. J. White paper ([16]). There we encounter the axiom $FA \land FB \rightarrow F(A \land B) \lor F(A \land FB) \lor F(FA \land B)$ that obviously requires a linear future.

References


Department of Logic
Nicolas Copernicus University
Toruń, Poland
e-mail: jarmuzek@unk.pl