BELIEF REVISION AND DOXASTIC COMMITMENT

A complex is a pair \((V, T)\) where \(V\) and \(T\) are theories in some given language and with respect to some given logic \(L\) (see [1] for definitions and assumptions). There is an operation \(*\) on complexes; it is the only primitive operation studied here, although of course further operations can be added. Informally, \(T\) is the old notion of a *belief set* and \(V\) the new notion of a *commitment set* – a set of doxastic commitments – while \(*\) is a revision operation. In this presentation, doxastic commitments are treated as irrevocable. Consider the following postulates (where the numbering has been chosen with an eye to the numbering in [1]):

\((*)a)\) If \((V, T)\) is a complex, then \(V \subseteq T\),
\((*)1)\) \((V, T) * \varphi = (V', T') \implies \varphi \in T'\),
\((*)b)\) \((V, T) * \varphi = (V', T') \implies V' = \text{Cn} (V \cup \{\varphi\})\),
\((*)2)\) \((V, T) * \varphi = (V', T') \implies V' = \text{Cn} (V \cup \{\varphi\})\),
\((*)3)\) \((V, T) * \varphi = (V', T') \implies \text{L-consistent} \implies T' = \text{Cn} (T \cup \{\varphi\})\),
\((*)4)\) \((V, T) * \varphi = (V', T') \implies \text{L-consistent} \implies T' = \text{L-consistent}\),
\((*)5)\) \((V, T) * \varphi = (V', T') \& (V, T) * \psi = (V'', T'') \implies (L \vdash \varphi \equiv \psi \implies T' = T'')\),
\((*)c)\) \(((V, T) * \varphi) * \psi = (V', T') \& (V, T) * (\varphi \land \psi) = (V'', T'') \implies T' = T''\).

Notice that \(T * \varphi = T'\) in AGM if and only \((\emptyset, T) * \varphi = (\text{Cn} \{\varphi\}, T')\) in our system. In this sense, our system is a generalization of the part of AGM that deals with revision. However, unlike AGM our system allows for iterated change.

A modelling for this theory along the lines of Lewis [3] and Grove [2] suggests itself. The operation of revision may be thought of as an operation \(*\) that transforms any belief state \(S\) and proposition \(P\) into a (usually but not necessarily different) belief state \(S * P\). If belief states are represented by systems of spheres over some given universe \(U\) (and thus propositions are certain subsets of \(U\)), then it is natural to take \(\cup S\) and \(\cap S\) as the semantic
representations of the commitment set and the belief set, respectively, and define

\[ S \ast P = \begin{cases} \{\emptyset\}, & \text{if } \cup S \cap P = \emptyset, \\ \{X \cap P : X \in S \text{ & } X \cap P \neq \emptyset\} \cup \{\emptyset : \emptyset \in S\}, & \text{otherwise.} \end{cases} \]

It is noteworthy that a belief state can change by revision by a proposition even if that proposition is already believed (but does not represent a commitment).

The proposed modelling is perhaps best studied with the help of DDL (dynamic doxastic logic; see [4, 5]). All but one of the postulates above-(*1) is special-have obvious counterparts in DDL (we write b for \( \neg B \neg \) and k for \( \neg K \neg \)):

1. \( K \varphi \supset B \varphi \),
2. \([*\varphi]B \varphi \),
3. \([*\varphi]K \chi \equiv K(\varphi \supset \chi)\),
4. \([*\varphi]B \chi \equiv B(\varphi \supset \chi) \),
5. \([*\varphi]B T \),
6. \([*\varphi]B \chi \equiv [\neg \varphi]B \chi \),
7. \([*\varphi]B \chi \equiv [\neg \varphi]B \chi \).

A complete axiomatization is given in [6].

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References


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