A SUMMARY OF THE UNDERLYING GROUND FOR HYPOTHETICAL PROPOSITIONS

In “The Underlying Ground for Hypothetical Propositions” by the author, Logic in the 20th Century, Scientia (1983) pp. 101–124, a range of logics are presented, where the lattice valuation of a hypothetical proposition \( X \supset Y \) is given by

\[
val(X \supset Y) = u \land (val(X) \mathcal{Q} val(Y)) \lor z.
\]

Let \( A \) be a bounded distributive lattice. Let \( Q \) be a sublattice of \( A \) containing 0, 1 such that for each \( x, y \) in \( A \) there is an element \( x \mathcal{Q} y \) in \( Q \) with the property that for all \( q \) in \( Q \), \( x \land q \leq y \) if and only if \( q \leq x \mathcal{Q} y \). The elements \( u, z \) may be construed as valuations of certain logical constants, the former being designated and the latter being antidesignated. Typically \( z \leq u \), with \( z \lor x = u \) implying that \( x = u \), after Axiom 2c in [2].

A hierarchy of logics arises through the cases (i) \( z = 0, \ u = 1 \); (ii) \( z = 0 \), general \( u \); (iii) general \( z \), general \( u \). Further, \( Q \) might be all of \( A \), or equal to the center of \( A \), or equal to the exocenter of \( A \) [4,5], to give 3 pivotal examples.

The first two cases (i), (ii) are related to logics with modal implications at the third or higher levels. Case (iii) is related to logics with minimal implications that diverge from weaker modal logics. Some minimal implications are described in [1]. Some aspects of Case (iii) are discussed in [3]. In particular,

\[
(A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C))
\]

does not hold in the weak logic systems of [3].

References


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