AN INTERPRETATION OF ARISTOTLE’S SYLLOGISTIC
AND A CERTAIN FRAGMENT OF SET THEORY IN
PROPOSITIONAL CALCULI

In [1] Chapter IV Łukasiewicz presents a system of syllogistic which
is an extension of Aristotle’s ordinary syllogistic¹. In spite of this differ-
ence Łukasiewicz speaks about it, as do we, as the Aristotelian system.
One of the well-known interpretation of syllogistic is Leibnitz’s interpre-
tation described in [1] (pp. 126–129). Syllogistic formulas are interpreted
there in an arithmetical manner. A second, very natural interpretation,
has been given by Slupecki (see below), who interprets syllogistic formulas
set theoretically. Although every formula which is not a syllogistic the-
esis can be rejected by using a finite number of objects (natural numbers
in Leibnitz’s interpretation, and sets in Slupecki’s interpretation), there is
not any fixed finite number of objects that would falsify every formula not
being a syllogistic thesis². Our first interpretation (comp. Theorem 1)
has the advantage of interpreting Aristotle’s syllogistic in a finite- (four-)
valued propositional calculus. We also give an interpretation of Aristotle’s
syllogistic and a fragment of set theory in the modal calculus S5.

Let $\text{Term}$ be an infinite set of terms of Aristotle’s syllogistic ($AS$). El-
lementary formulas of $AS$ are expressions of the following forms: $\delta a b, \iota a b$

¹As is known, Aristotle’s syllogisms have the form $A \land B \rightarrow C$ where $A, B, C$ are
formulas (propositions) on a strictly defined form (comp. textbooks and also [2]). But
in the system presented by Łukasiewicz all formula are considered which can be formed
by means of categorial formulas and classical connectives (see below the definition of
the set of syllogistic formulas $AS$ and Note 3).

²In the syllogistic language it is possible to express a formula $A$ which is a disjunction
of the following set of formulas: $\{ \delta a_i a_j : 1 \leq i, j \leq k + 1 \}$ (we read $\delta a_i a_j$ as: every $a_i$
is $a_j$). This is not a syllogistic thesis. If, now, in some interpretation would be allowed
using only at most $k$ objects (i.e. $k$ natural numbers in Leibnitz’s interpretation and $k$
sets in Slupecki’s interpretation), then formula $A$ would not be rejected. This can be
seen especially well in Slupecki’s interpretation, where formula $A$ can be read as follows:
among every $k + 1$ sets there are at least two such sets that one includes the second.
(a, b ∈ Trm) which can be read: ‘every a is b’, ‘some a are b’ respectively\(^3\).

Complex formulas of \(AS\) are formed in the usual manner by means of elementary formulas and classical connectives: \(\neg\) (negation), \(\land\) (conjunction), \(\lor\) (disjunction), \(\rightarrow\) (implication) and \(\equiv\) (equivalence).

In order to attain our aim it is not necessary to present the system \(AS\) exactly. It will be enough if we describe only the method of deciding when a formula is or is not an \(AS\)-thesis. In [2] in the proof of Theorem IV (pp. 23–25) Slupecki shows that if terms from \(Trm\) are treated as variables ranging over nonempty sets and if the expressions of types \(A_{ab}\), \(I_{ab}\) are understood respectively as the expressions of the types: \(\lceil a \subseteq b \rceil\), \(\lceil a \cap B \neq \emptyset \rceil\) then we will have a method which enables us to decide whether a formula is or is not an \(AS\)-thesis.

Łukasiewicz shows (cf. [1] p. 120) that we are able to decide whether a formula is an \(AS\)-thesis or not iff we are able to decide whether a certain finite set of formulas determined by this formula, of the form:

\[
A_1 \rightarrow (A_2 \rightarrow \ldots \rightarrow (A_{k-1} \rightarrow A_k) \ldots)
\]

where \(A_1, \ldots, A_k\) (\(k = 1, 2, \ldots\)) are elementary formulas or their negations, is or is not a set of \(AS\)-theses. It can easily be seen that every formula of this kind is equivalent to some formula of the following form:

\[
(\ast) \ A_1 \land \ldots \land A_m \rightarrow B_1 \lor \ldots \lor B_n
\]

where \(m, n = 0, 1, \ldots; m+n > 1\) and \(A_1, \ldots, A_m, B_1, \ldots, B_n\) are elementary formulas (if \(m = 0\), then the considered formula has the form: \(B_1 \lor \ldots \lor B_n\), and if \(n = 0\) then our formula has the form: \(\sim (A_1 \land \ldots \land A_m))\). Thus the problem of the interpretation of Aristotle’s syllogistic can be reduced to the interpretation of formulas of the form \((\ast)\).

Let \(Frm\) be a set of formulas formed in the usual manner by means of propositional variables from their infinite set \(Vr\), and two argument connectives \(\bar{A}\) and \(\bar{I}\). \(Vr(P) (P \in Frm)\) denotes the set of propositional variables which the formula \(P\) contains. Consider the following matrix:

<table>
<thead>
<tr>
<th>(\bar{A})</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>*1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>*2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>*3</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

\(\bar{I}\) |
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0 | 0 | 1 | 1 | 1 |
1 | 1 | 1 | 1 | 1 |
2 | 2 | 1 | 1 | 2 |
3 | 3 | 1 | 1 | 0 | 3 |

\(\ast\) Usually, syllogistic contains also elementary formulas of the form: \(E_{ab}(= \sim I_{ab})\) and \(O_{ab}(= \sim A_{ab})\).
where the asterisk * indicates distinguished elements of the matrix. In what follows the notion of tautology is related to this matrix, we have assumed that the sets $Trm$ and $Vr$ are infinite. For simplicity we will assume that they have the same cardinality and that the function $f : Trm \rightarrow Vr$ establishes this property. Let $*$ be a function on elementary formulas such that:

$$(Aab)^* = \bar{A}f(a) f(b)$$

$$(Iab)^* = \bar{I}f(a) f(b).$$

The letter $e$ will denote substitutions in $Frm$, i.e. $e$ is a function such that $eVr \subseteq Frm$ and for all $P, Q \in Frm$, $e(APQ) = \bar{A}ePeQ$ and $e(IPQ) = \bar{I}ePeQ$.

**Theorem 1.** Let $A_1, \ldots, A_m, B_1, \ldots, B_n$ be elementary formulas of $AS$ ($m, n = 0, 1, \ldots$ and $m + n \geq 1$). $A_1 \wedge \ldots \wedge A_m \rightarrow B_1 \lor \ldots \lor B_n$ is an $AS$-thesis iff for every substitution $e$ if all formulas $e(A_1^*), \ldots, e(A_m^*)$ are tautologies then at least one of the following formulas: $e(B_1^*), \ldots, e(B_n^*)$, is a tautology.

As we mentioned for interpretation of Aristotle’s syllogistic it is sufficient to interpret formulas of type (*). Therefore Theorem 1 really establishes an interpretation of $AS$-syllogistic in propositional calculus determined by the above 4-valued matrix.

Let the symbols $M, L$ denote the connectives of possibility and necessity respectively, and let $T$ be an interpretation such that:

$$TA = Mf(a_1) \wedge \ldots \wedge Mf(a_k) \rightarrow tA$$ where $a_1, \ldots, a_k \in Trm$ are all variables of the formula $A$ and

$$t(Aab) = (a, b \in Trm)$$

$$t(Iab) = M(f(a) \wedge f(b))$$

$t$ preserves all classical connectives.$^4$

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$^4$We assume, as above, that the function $f$ establishes the equality of cardinalities of $Trm$ and the set of propositional variables of the language of $S5$. 

The theorem establishing the connections between AS-syllogistic and prepositional calculus $S_5$ is the following:

**Theorem 2.** A formula $A$ is an AS-thesis iff $TA$ is an $S_5$-thesis.

The final point is an interpretation of some fragment of set theory in the modal calculus $S_5$. We will interpret a fragment of the set theory, which we will denote by the symbol $FST$, containing all formulae built by means of variables ranging over sets (their set will be denoted by the symbol $Trm$), operations: $\neg$, $\cap$, $\cup$; the set $\emptyset$, relation $\subseteq$ and the classical connectives. The set of expressions built by means of variables from $Trm$ and symbols: $\neg$, $\cap$, $\cup$ will be denoted by the symbol $Trm^+$. $s$ is the interpretation preserving the classical connectives and satisfying the following conditions:

$$
\begin{align*}
sa &= f(a) \\
\left(\alpha - \beta\right) &= s\alpha \land \sim s\beta \\
\left(\alpha \cap \beta\right) &= s\alpha \land s\beta \\
\left(\alpha \cup \beta\right) &= s\alpha \lor s\beta \\
\left(\alpha \subseteq \beta\right) &= L(s\alpha \rightarrow s\beta) \\
\emptyset &= \sim (f(a) \rightarrow t(a)), \text{ where } a \in Trm \text{ can be any think but fixed.}
\end{align*}
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**Theorem 3.** A formula $A \in FST$ is a theorem of set theory iff $sA$ is an $S_5$-thesis.

**References**


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5The system of set theory which we consider is the system of Zermelo-Fraenkel.