DEFINABILITY CRITERION FOR FUNCTIONS IN SUGIHARA ALGEBRAS

This is an abstract of the second part of a paper that will be published in *Studia Logica*.

Let \( A = (A, \sim, \lor, \land, \rightarrow) \) be a Sugihara algebra (see [1]). For \( a \in A \), we shall write \( a = 0 \) instead of \( \sim a = a \).

**Definition 1.** Two sequences \( a = (a_1, \ldots, a_k) \), \( b = (b_1, \ldots, b_k) \) of elements of \( A \) are said to be EXTREMALLY SIMILAR, in symbols \( a \simeq b \), provided that for all \( i, j \), \( 1 \leq i, j \leq k \),

\[
\begin{align*}
(i) & \quad a_i = 0 \text{ iff } b_i = 0; \quad a_i < 0 \text{ iff } b_i < 0, \\
(ii) & \quad |a_i| \leq |a_j| \text{ iff } |b_i| \leq |b_j|.
\end{align*}
\]

Let \( f \) be a variable ranging over \( k \)-ary functions on \( A \), that is \( f : A^k \to A \).

**Definition 2.** A function \( f \) is said to PRESERVE SIMILARITY provided that for every \( a, b \in A^k \), if \( a \simeq b \) then for all \( i \), \( 1 \leq i \leq k \),

\[
\begin{align*}
(i) & \quad f(a) = a(i) \text{ iff } f(b) = b(i) \\
(ii) & \quad f(a) = \sim a(i) \text{ iff } f(b) = \sim b(i).
\end{align*}
\]

**Theorem.** A function \( f \) is definable (see [1]) in \( A \) if and only if

\[
\begin{align*}
(i) & \quad \text{for every } a = A^k, f(a) \in \{a(1), \ldots, a(k), \sim a(1), \ldots, \sim a(k)\}, \\
(ii) & \quad f \text{ preserves similarity.}
\end{align*}
\]
References